Irrigation control: towards a new solution of an old problem
Irrigation control: towards a new solution of an old problem

Gerd H. Schmitz,
Niels Schütze and
Thomas Wöhling

Koblenz 2007
A German contribution to the theme Water and Food of the programme Hydrology for the Environment, Life and Policy (HELP), a joint initiative of UNESCO and WMO.

Authors:

Gerd H. Schmitz  
Institute for Hydrology and Meteorology, Dresden University of Technology, Germany

Niels Schütze  
Technion – Israel Institute of Technology, Haifa, Israel

Thomas Wöhling  
Lincoln Environmental Research, Hamilton, New Zealand

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© IHP/HWRP-Sekretariat  
Bundesanstalt für Gewässerkunde  
Postfach 200253  
56002 Koblenz, Deutschland  
Fax  +49 (0)261 1306 5422
One of the most challenging tasks of the 21st century is securing the food supply for the growing world population. In the coming years, the global food production will have to increase by approximately 40% to meet the main objective of the UN Millennium Development Goals, that is to reduce by half the proportion of people who suffer from hunger.

In this context, the crucial and restrictive factor of all food production activities, be it livestock breeding or crop farming, is water. At least 70% of the global freshwater consumption is used for irrigational purposes. The water supply for irrigation competes with other growing demands such as domestic and industrial water consumption. An effective and sustainable management of irrigation in order to optimise the food production will play a key role in the fight against hunger. This is portrayed tellingly by the 1st United Nations World Water Development Report, published in 2003, which urgently calls for "more crop per drop".

Irrigation has many facets.Rainfall, temperature and the soil conditions are amongst the many parameters which have to be taken into account for irrigation scheduling. Water has to be dosed to ensure optimal growing conditions for crops. Erratic irrigation might lead to sub-optimal plant growth due to stress caused by drought or the noxious inundation of plant roots. Furthermore, soils are sensitive to leaching caused by excessive irrigation. Preventing salinisation due to evapotranspiration is another main concern of sustainable irrigation management. All these factors and site-specific conditions interact and set the frame for the task of optimising irrigation strategies.

Farmers need a flexible, reliable and robust means for optimising their irrigational practices, especially in the context of the many variables and conditions that affect irrigation. After comprehensively laying out the foundations of irrigation, this study offers a tool for operational irrigation planning. It is based on considering the physical processes dominating the infiltration of water into the soil, plant water uptake and evapotranspiration. These complex processes are portrayed by a tailor-made neural network solution, enabling farmers to use efficient operational irrigation control without in-depth knowledge of the complex underlying physics.

This study is a contribution to the theme Water and Food of the programme Hydrology for the Environment, Life and Policy (HELP). HELP is a joint initiative of UNESCO and WMO, and is led by the International Hydrological Programme (IHP) of UNESCO.

With the publication of Heft 5 in the IHP/HWRP-Berichte series with the title *Irrigation control: towards a new solution of an old problem*, the German IHP/HWRP National Committee would like to make a contribution towards an improved efficiency of irrigated agriculture. We thank the authors for their excellent work and valuable contribution.

Gerhard Striegel
Acting Director of the German IHP/HWRP Secretariat
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<td>AD</td>
<td>adequacy</td>
</tr>
<tr>
<td>AE</td>
<td>application efficiency</td>
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<tr>
<td>AI</td>
<td>artificial intelligence</td>
</tr>
<tr>
<td>ANN</td>
<td>artificial neural networks</td>
</tr>
<tr>
<td>ASCE</td>
<td>American Society of Civil Engineers</td>
</tr>
<tr>
<td>BMU</td>
<td>best matching unit</td>
</tr>
<tr>
<td>CEF</td>
<td>closed-end furrow</td>
</tr>
<tr>
<td>CPN</td>
<td>counter propagation network</td>
</tr>
<tr>
<td>CPU</td>
<td>central processing unit</td>
</tr>
<tr>
<td>DE</td>
<td>differential evolution</td>
</tr>
<tr>
<td>DP</td>
<td>dynamic programming</td>
</tr>
<tr>
<td>DU</td>
<td>distribution uniformity</td>
</tr>
<tr>
<td>EA</td>
<td>evolutionary algorithm</td>
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<tr>
<td>ET</td>
<td>evapotranspiration</td>
</tr>
<tr>
<td>ETP</td>
<td>potential evapotranspiration</td>
</tr>
<tr>
<td>FAO</td>
<td>Food and Agriculture Organization</td>
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<tr>
<td>FDF</td>
<td>free-draining surface flow</td>
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<tr>
<td>FIDO</td>
<td>furrow irrigation design optimizer</td>
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<tr>
<td>FIM</td>
<td>furrow irrigation model</td>
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<tr>
<td>GA</td>
<td>genetic algorithm</td>
</tr>
<tr>
<td>GAIN-P</td>
<td>genetic algorithms, artificial intelligence techniques and process modelling</td>
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<tr>
<td>GUI</td>
<td>graphical user interface</td>
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<tr>
<td>HD</td>
<td>hydrodynamic</td>
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<tr>
<td>HI</td>
<td>harvest index</td>
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<tr>
<td>IBMU</td>
<td>interpolation using the centre of gravity of the $n$ best matching units</td>
</tr>
<tr>
<td>IE</td>
<td>irrigation efficiency</td>
</tr>
<tr>
<td>IHP</td>
<td>International Hydrological Programme</td>
</tr>
<tr>
<td>IIT</td>
<td>Indian Institute of Technology (Kharagpur, India)</td>
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<td>ITRI</td>
<td>interpolation with triangulation networks</td>
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<tr>
<td>KW</td>
<td>kinematic-wave</td>
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<tr>
<td>LAI</td>
<td>leaf area index</td>
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<tr>
<td>LDV</td>
<td>Laser Doppler Velocity</td>
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<tr>
<td>MAD</td>
<td>management allowable depletion</td>
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<tr>
<td>MLP</td>
<td>multilayer perceptron network</td>
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<tr>
<td>OCR</td>
<td>optical character recognition</td>
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<tr>
<td>RBF</td>
<td>radial basis function network</td>
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<tr>
<td>RUE</td>
<td>radiation use coefficient</td>
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<tr>
<td>SA</td>
<td>simplex annealing</td>
</tr>
<tr>
<td>SCE</td>
<td>shuffle complex evolution</td>
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<td>SI</td>
<td>sensitivity index</td>
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### Irrigation control: towards a new solution of an old problem

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<th>Definition</th>
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<td>SMD</td>
<td>soil moisture deficit</td>
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<tr>
<td>SOLO</td>
<td>self-organizing linear output mapping network</td>
</tr>
<tr>
<td>SOM</td>
<td>self-organizing map</td>
</tr>
<tr>
<td>SOM-MIO</td>
<td>self-organizing map with multiple input/output option</td>
</tr>
<tr>
<td>STI</td>
<td>sand tube irrigation</td>
</tr>
<tr>
<td>TDR</td>
<td>time domain reflectometry</td>
</tr>
<tr>
<td>VB</td>
<td>volume-balance</td>
</tr>
<tr>
<td>WRI</td>
<td>Water Resources Institute</td>
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<tr>
<td>ZI</td>
<td>zero inertia</td>
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</table>
Preface

Irrigation efficiency and sustainability are of paramount importance when it comes to securing the food supply for future generations. Unfortunately, the most widespread traditional irrigation methods generally only manage to achieve poor irrigation efficiencies and long-term sustainability aspects continue to be disregarded.

A highly efficient, yet at the same time sustainable, irrigation needs to exploit the planning and operative potentialities provided by innovative, more advanced technologies. Today, computer-based technology already plays an important role in identifying, comparing and/or assessing the sustainability of water resources management in irrigation. These modern techniques, however, are only applied in relatively few cases. Unfortunately, current practice still employs outdated, simplistic tools which do not really allow for a physically sound consideration of the interacting water transport processes from the field entrance to the roots of the plants and down to the groundwater level. This applies even more for the related optimization problem, i.e. when water application and scheduling parameters have to be evaluated in an attempt to achieve an optimal field irrigation efficiency yet still maintaining the sustainability of the system.

Only simulations with sound and reliable irrigation models can predict the future impacts of any proposed irrigation strategy and/or management policy. Up until now, physically based process modelling was the best tool for dealing with problems such as changes in the irrigation system or with extrapolations beyond the range of the calibration experiments, or when investigating the consequences of global change scenarios. Unfortunately, the big drawback of these coupled numerical irrigation models lies in their extremely high computational effort. Furthermore, their operation may encounter numerical instabilities and requires a comprehensive experience in numerical modelling which, together, may be the reason why rigorous process modelling never came to be a standard tool in irrigation practice. The alternative approach, i.e. simplistic empirical models, satisfy the requirements of easy application and low execution times, but they are not able to provide sound and comprehensive results particularly when working beyond the calibration range of a few field experiments. This is generally the case when investigating irrigation scenarios or applying optimization strategies. An intense field experimentation is not the answer here either because this would involve an inordinate amount of time and financial investment. As regards the currently available methods and strategies, we thus find ourselves between a rock and a hard place.

Traditional control in furrow irrigation
But is there an alternative? We need simple, robust models capable of being used by unskilled personnel and able to make prognostic assessments of soil water transport processes and which are, furthermore, capable of defining these in a high temporal and spatial resolution. The second problem to arise when developing planning tools for a highly efficient and sustainable irrigated agriculture is that numerous objectives have to be satisfied simultaneously. Besides the economic objectives there are the ecological and social ones, too. The broader range of objectives necessitates not only more complex models for portraying all the different aspects, it also leads straight to optimization problems which are more difficult to solve than those we had to deal with in the past. The optimization tools currently available are insufficient for successfully tackling this new generation of optimization problems.

This contribution introduces a new approach for the planning and control of highly efficient yet sustainable irrigation systems. The new approach is founded firstly upon physically based modelling and secondly upon the organizational patterns of complex natural systems such as the human brain or evolution processes: artificial neural networks (ANN) and evolutionary algorithms (EA). The subsequent development and application of a comprehensive irrigation modelling approach – on the basis of ANN and EA – demonstrates its ability for solving the technical problems arising from the simulation-based control and scheduling of irrigation systems.

In the first approach, the artificial neural network portrays a well-established practice used in traditional irrigation techniques. In areas where irrigated agriculture has a long tradition and has given rise to a certain social continuity, the application of the approach and the ensuing continual improvement of the traditional irrigation methods were observed to bring very positive results. This old trial and error strategy is replaced by simulating numerous irrigation cycles using physically based models together with artificial neural networks. The latter acquire extensive knowledge, i.e. the optimal controlling of an irrigation plant by analyzing in fast forward modus the information fed to them during a long (simulated) period of observation. On this basis, relatively low maintenance irrigation systems can be set up reliably and efficiently, without the need for operative numerical modelling.

The second new approach demonstrates how evolutionary algorithms can assist in attaining the goal of a sustainable irrigated agriculture. The basic mechanisms of natural evolution – mutation, selection and recombination – are exploited with the help of a physically based irrigation model for calculating irrigation patterns, which can respond to the various objectives and requirements of a sustainable management. The evolutionary algorithm finds the close-to-optimal solutions within a surprisingly short time and tends to be very robust and versatile, in that it can be applied to a wide range of optimization problems in water resources management.

By combining a sound, physically based irrigation model with evolutionary algorithms and artificial neural networks, simple and reliable control and scheduling parameters can be calculated to ensure a sustainable management in the irrigated agriculture. A seasonal furrow irrigation model (FIM) is presented, which comprises process-based simultaneous modelling of the 1D surface flow, the quasi-3D soil water transport and the crop growth. FIM is quite flexible and able to be adapted for many specific demands. By putting the new approaches into operation, the personnel responsible for irrigation and accustomed to working with simple tools will quickly be able to reap the benefits of the improved irrigation performance thanks to the physically based models.

After having shown how tailor-made models are able to simulate all the essential processes involved in water transport throughout the entire growth period and after showing how to
combine these models with the new optimization methods, we will further demonstrate in this contribution practically relevant application examples for operative irrigation planning in furrow and micro irrigation. We aim to show that new complex management tools must not automatically damage the long-established social and historical characteristics of a given area, e.g. the traditional irrigation techniques and the educational standards. We see here that new technical developments can be adapted to suit the farmers' very basic level of education and acquired skills and that it is not necessarily the case that each new technological development inevitably brings about negative social change.

In the light of this, the subsequently presented strategy hopes to contribute towards achieving the overall goal of "more crop per drop", by focusing on the weakest link in the irrigation efficiency chain, namely, the water application. In contrast to, for example, the conveyance efficiency, the water application efficiency of the most common irrigation methods is still astonishingly poor. Apart from socio-economic aspects such as water pricing, traditional water rights, etc., the reason for the poor irrigation efficiency seems to be the difficulty in finding an adequate answer to the million-dollar question: with respect to the most efficient and sustainable water use, what are the best water application AND scheduling parameters? This question leads to an extremely complex optimization problem. Moreover, its solution requires a comprehensive description of the following cause-and-effect principle: which crop best responds to which combination of water application and scheduling parameters, and vice versa, which combination of the parameters is best suited for which crop? A comprehensive relationship between irrigation strategy and crop yield can be provided by a sound, physically based portrayal of the relevant water transport processes in and above the soil. Such process modelling goes hand in hand with numerical methods which, unfortunately, have the disadvantage of a complex and cumbersome operation along with extensive computational requirements, especially in terms of execution time. These negative aspects restrict their combined use with current techniques for optimizing irrigation parameters. An even more serious constraint is that irrigation practice requires simple, robust tools and, in general, cannot cope with highly sophisticated numerical modelling together with nonlinear optimization procedures. The subsequently outlined innovative principle of optimally evaluating water application parameters and irrigation scheduling parameters GAIN-P (Genetic evolutionary algorithm, Artificial INtelligence and Process modelling) offers a way out of the dilemma which arises from the need for a reliable, predictive simulation and optimization tool, yet at the same time the need for a simple tool which is straightforward to operate.

Professor Dr G.H. Schmitz
Dresden University of Technology
Introduction

The world's population has more than doubled in the last half century and topped six billion in 1999. By 2030, it is projected to reach around eight billion and nearly all of that increase is expected to occur in developing countries. The demand for food increases with population growth and the ensuing increase in agricultural production places ever greater demands on water. Especially in developing countries, the area equipped for irrigation is expected to have expanded by 20% (i.e. an additional 40 million hectares) by 2030. This means that 20% of the total land with irrigation potential – but not yet equipped – will be brought under irrigation and that 60% of all land with irrigation potential (i.e. a total of 402 million hectares) will be in use by 2030. The Food and Agriculture Organization (FAO) predicts that agricultural water withdrawals will increase by some 14% from 2000 to 2030 in order to satisfy future food production needs [UNESCO, 2003].

As the world's population continues to grow, the consumption of water in all sectors of water use (agricultural, industrial and domestic) increases accordingly. Consequently, water will soon become even scarcer and more expensive, requiring high use efficiencies and precise water management. Agriculture is still the greatest water user of all\(^1\) while having the lowest water use efficiency\(^2\). Especially irrigated agriculture is particularly guilty of inefficient water use, the pollution of ground and surface water, and land degradation. Thus, good water management practices in irrigation aim to improve water use efficiency, along with preserving the soil and water resources, without sacrificing crop productivity. In addition, water resources management should ensure a sustainable, long-term improvement in the quality of life and natural systems. For the above reasons it has become evident that many of the current water resources management practices have to be re-examined and new tools for sustainable water management need to be found.

The American Society of Civil Engineers (ASCE), associated with the United Nations International Hydrological Programme (UN/IHP), published a monograph on sustainable water resources management [Loucks and Gladwell, 1999]. According to the report, sustainable water resource systems are those designed and managed to fully contribute to the objectives of society, now and in the future, whilst maintaining their ecological, environmental and hydrological integrity. The guidelines for sustainable water resources management can be briefly outlined as follows:

- to achieve financial and economic efficiency
- to minimize long-term cumulative negative environmental impacts

\(^1\) At the start of the 21\textsuperscript{st} century, agriculture is responsible for a global average of 70% of all water withdrawals from rivers, lakes and aquifers [Water Resources Institute (WRI), 2005].

\(^2\) In irrigation terms it is estimated that the overall water use efficiency in developing countries is about 38% [UNESCO, 2003].
Irrigation control: towards a new solution of an old problem

- to maintain stability and flexibility in the water supply, i.e. to deal with extreme events such as drought and other anticipated stochastic events
- to incorporate the most important features of the local social situation
- to perform reliably and be able to adapt to new technology.

Computer-based modelling technology plays a critical role in identifying and comparing or assessing the sustainability of water resources management in irrigation. Without simulation models, it would be difficult to predict the future impacts of any proposed plan and management policy. Thus, the management and operation of sustainable irrigation systems depends on the state-of-the-art modelling technology, as well as on adequate optimization tools. There are obviously uncertainties and question marks with respect to future climate change and its inestimable global effects; physically based process models are by far the best equipped to assess possible future developments and their consequences. This contribution proposes tailor-made models for furrow and micro irrigation systems. These are in a position to simulate all the essential processes involved in water transport throughout the entire growth period.

A seasonal furrow irrigation model (FIM) is presented which comprises process-based simultaneous modelling of the 1D surface flow, the quasi-3D soil water transport and the crop growth. FIM is quite flexible and able to be adapted for many specific purposes. Besides the physically based modelling of traditional furrow irrigation, we also deal with modern irrigation practices. The currently available models for describing water movement on field scale and for subsurface flow can unfortunately only in part satisfy the demands placed on them by irrigation practice and system sustainability. What we need are simple, robust and computationally efficient models, which are able to simulate the irrigation process in a high temporal and spatial resolution. Although process-based modelling allows for a high temporal and spatial resolution with respect to prognostic computations, it unfortunately requires an extremely high computational effort. Moreover, these physically based numerical models do not always perform in an unconditionally stable manner due to discontinuous flow processes. These disadvantages of process-based modelling are a fundamental drawback when dealing with optimization procedures. This is because combining process modelling with traditional optimization strategies automatically leads to far more complex and instable algorithms.

It is extremely difficult to use process-based modelling for solving the type of multiple objective optimization problems which typically arise when dealing with sustainable water resources management. Because the target function has many locally optimal solutions and often features an undefined number of optimization variables, it is often impossible to guarantee convergence or find the best solution in an acceptable amount of time. Thus, up until now, process-based sustainable water resources management systems have not been employed in irrigation practice. Standard procedure continues to use empirical descriptions of the relevant flow processes.

In this monograph we present and analyze the fundamentally new methodology GAIN-P also with respect to contrasting applications of current planning irrigation and control strategies. We do this on the basis of physically based process modelling and with new optimization methods originating from the field of Evolutionary Algorithms (EA) and Artificial Intelligence (AI). The new strategy utilizes problem-adapted Artificial Neural Networks (ANN) which are able to accurately portray the functioning of comprehensive physically based irrigation models, with respect to a considered area. In contrast with other currently used approaches, the new strategy combines the advantages of ANNs, i.e. robustness and high computational efficiency, with the advantages of physically based modelling, namely, a detailed process description. At the same
time, the new approach overrides the disadvantages of current methods, e.g. high computational effort and very limited reliability. Moreover, the new management tools are extremely easy to operate and do not require numerical expertise.

In addition, a tailor-made evolutionary algorithm is used to solve complex optimization problems which arise when multiple objectives have to be attained simultaneously for a sustainable irrigation scheduling. Evolutionary algorithms provide near-optimal solutions within a reasonable amount of time and tend to be very robust and versatile. They can thus be applied to a wide range of optimization problems in water resources management.

By combining process modelling with evolutionary algorithms and artificial neural networks, we are presented with a new generation of management tools for practical application. This new generation builds on a sound process description, is able to simply and reliably define the appropriate controlling and scheduling parameters and, furthermore, can ensure a sustainable efficiency for irrigated agriculture.

This contribution begins with an introduction of the basic terms in irrigation scheduling and stresses the components which play a role in the simulation-based management of irrigation systems. In section 1.3 there is a résumé of the currently used approaches in the areas of irrigation modelling and simulation-based control and scheduling. This discloses the deficiencies of currently used approaches, especially with respect to optimal control and scheduling for achieving an efficient and sustainable irrigation system.

In Chapter 2, a symbiosis between physically based irrigation modelling and artificial intelligence approaches (artificial neural networks and evolutionary algorithm) leads to a new strategy for irrigation management GAIN-P, using either deficit or full irrigation methods.

Chapter 3 introduces the basic principles and innovative developments with respect to (i) seasonal irrigation modelling, (ii) artificial neural networks and (iii) evolutionary algorithms, which altogether form the new simulation-based management tool.

Chapter 4, which deals with the application of the presented GAIN-P methodology, demonstrates how the new management tool can achieve improved irrigation efficiency with respect to furrow or micro irrigation.

The monograph concludes with Chapter 5, which highlights some key points concerning the planning and management of sustainable irrigation systems. This chapter also discusses the prospects for adapting the presented management strategy for more efficient and sustainable irrigation systems in the future.

### 1.1 Irrigation scheduling

Whichever type of irrigation system is used, it is important to use some method of irrigation scheduling for determining the exact amount of water to apply to the field and the exact timing for application during a growing season. There are two basic methods of irrigation scheduling:

- sensor-based scheduling
- simulation-based scheduling.

Sensor-based scheduling relies on measurements taken from one or several of the indicators which monitor the irrigation requirements at the location. The climate, the plants themselves
or the soil can function as indicators. An irrigation event takes place when the monitored values exceed upper or lower boundary levels. Sensor-based scheduling is, then, a real-time procedure as far as irrigation control is concerned. However, in real terms, it is generally not foreseeable when a future irrigation will become necessary.

This is where simulation-based scheduling can help as it enables a sufficiently accurate prediction of future irrigation requirements. It does this by using a mathematical model to simulate the essential water transport processes for the irrigation system. On this basis, the necessary parameters can be calculated for all future irrigation events.

1.1.1 Sensor-based scheduling

The most common irrigation criteria are soil moisture content, soil water tension, evapotranspiration or transpiration rate and plant parameters like leaf water potential.

Figure 1 Monitoring soil water tension with tensiometers

Soil moisture content
The oldest and probably most widely used parameter for irrigation scheduling is the degree of permissible depletion of soil water. This determines what fraction of stored soil water is allowed to be depleted between irrigations. There are several tools available for measuring soil moisture content. Time Domain Reflectometry (TDR) is a common tool for soil water monitoring. It is portable and quick. Some units require calibration and may be finicky to work with.

Soil water tension
The monitoring of soil water tension informs us when moisture stress occurs and, thus, when irrigation should begin. Tensiometers measure how strongly water is being retained by the soil. As the soil dries out, moisture is less available to the crop due to the increase of the negative soil water pressure, i.e. the water tension or suction head. However, tensiometers can only be used to determine tensions up to 1 atm. Electrical Resistance Blocks also measure soil water tension in centibars. Compared to tensiometers, they are easier to install: the blocks are portable and inexpensive and are thus well-suited for soil moisture readings at different sites.
Evapotranspiration rate
Once the availability of soil water is reduced, the rate of evapotranspiration will decrease and this change in rate can be used for determining the next irrigation. This is less labour-intensive and is less subject to spatial variability than the determination of soil moisture content or tension. However, methods which determine evapotranspiration (ET) under limited water availability are not always reliable and are not sensitive enough to detect crop water stress.

Plant parameters
The most meaningful plant parameter for irrigation scheduling is the leaf water potential. Its determination will most likely involve the use of the pressure chamber technique. Although this method may not represent the actual leaf water potential, it is preferable to other more reliable methods which need more equipment and more constant environment conditions.

Each of the sensor-based control methods shows that an irrigation event only takes place as a reaction to a past action (e.g. the exceeding of a boundary level). The methods based on evapotranspiration rate and leaf water potential suffer from a substantial time-delayed reaction. For this reason, sensor-based scheduling is not suitable for irrigation control where future developments in the irrigation system are intended. It can only be applied at locations where there is a lot of local experience regarding the irrigation characteristics of the area.

1.1.2 Simulation-based irrigation scheduling with water budget methods
With simulation-based irrigation control and scheduling, the interactions of the various individual components involved in the soil-plant-atmosphere system are simulated for the duration of an entire growing period. For each time step \( t \) \((t = 1d)\) during the considered growth period, the changes in the soil water reservoir are defined using a water balance equation:

\[
S_e(t) - S_o(t) = I(t) + P(t) + UP(t) - DP(t) - E(t) - T(t) - RO(t). \tag{1}
\]

The parameters of the water balance in Equation 1 represent the results of the interacting dynamic processes involved in water transport. Generally speaking, these are: precipitation \( P \), evaporation \( E \), plant transpiration \( T \), the irrigation process \( I \), encompassing a possible runoff \( RO \). The soil water transport induces changes in the water content of the soil \( S \) and leads to phenomena such as deep percolation \( DP \) or capillary rise \( UP \). Regrettably, the currently used water balance models do not take into account the most crucial phenomena with respect to a sustainable irrigation; namely, they neglect the capillary rise and sometimes even the runoff resulting from (over) irrigation and precipitation.

The water budget method is simply an accounting procedure similar to the bookkeeping required to balance a bank account. If the balance on a given date and the amounts of the transactions are known, the balance can be calculated at any time. For irrigation scheduling, soil water content is balanced. The amount of water that is lost as crop evapotranspiration is analogous to making a cash withdrawal. The water that enters the soil reservoir (as rain or irrigation) is analogous to depositing funds in the account. By keeping records of these transactions, it is possible to know how much water is in the soil reservoir at anytime. By simulating the water balance throughout a growth phase, the water requirements of an irrigation system can be estimated and an appropriate guess of an irrigation schedule, i.e. when and how much to irrigate, can be set up.
Irrigation control: towards a new solution of an old problem

Water balance models
As far as the control and planning of irrigation projects is concerned, it is general practice to use water balance models for the simulation of the water content in the soil-plant-atmosphere system [Smith, 1992; Rao and Sarma, 1990; Singh et al., 1995; Raghuwanshi and Wallender, 1997a]. The field is considered as a soil water reservoir; it has a certain capacity for water retention and depending on the situation and the prevailing climatic conditions can either take up or give off water from its available stock. The storage capacity of the field changes over time as a consequence of the plants' growth and the increasing root penetration. Depending on the characteristics of the considered soil type and the usable field capacity, the stored water is available for plant uptake. Irrigation is timed in accordance with a Management Allowable Depletion (MAD). This is the percentage of the available water which the farmer will allow plants to deplete before starting to irrigate or, alternatively, the depth of water that the farmer will allow plants to extract from the root zone between irrigations. The rough simplifications of the water balance models – i.e. neglecting to take either the spatial extension of the soil or the soil water dynamics into account – only allow for a rough estimation of the percentage of the water supplied by irrigation or precipitation which is actually used by the plants. Moreover, the computation of the components of the water balance equations ET, S, DP and UP are extremely vague due to the empirical character of the models employed.

Hydrodynamic water transport models
For a given irrigation site, dynamic process models provide information on soil water transport with a high temporal and spatial resolution. This provides, for example, information on soil moisture distribution, on the distribution of the hydraulic potential and the flow velocity patterns of the soil water. This results in a much more rigorous evaluation of the field water balance (as compared to the one provided by simple water balance modelling) and, thus, renders the computation of the irrigation schedule much more reliable. In addition, dynamic process models allow for the simulation of each single irrigation event and for a rigorous evaluation of the corresponding irrigation efficiency. Unfortunately, these types of models require numerical solution procedures (e.g. FEM or FDM) and can only be solved iteratively. This involves the risk of numerical instability or unsatisfactory convergence when evaluating the solution. It is also accompanied by a considerable computational effort and requires an above average amount of expertise in numerical modelling. The water budget method aims at fully satisfying the plant water requirements over a whole growing period, i.e. no restrictions are placed on the amount of water available. For this reason, this type of irrigation is also termed 'full irrigation'.

1.1.3 Optimization-based scheduling
When irrigation is constrained by limited water availability or limited irrigation system capacity, a maximum crop yield is not achievable. This so-called 'deficit irrigation' aims at finding an optimal irrigation schedule under which crops can sustain an acceptable degree of water deficit and yield reduction. An optimization method determines the most favourable irrigation schedule with the optimum trade-off between the volume of applied water and the yield to be achieved.

Irrigation optimization involves (i) the modelling of the water flow in irrigation, (ii) the specification of an objective function and (iii) a search technique of some sort which identifies optimal irrigation schedules on the basis of simulations (see Figure 2). Optimization-based scheduling is the subject of this monograph and is treated comprehensively in section 2.3.
1.2 Irrigation control

No matter which irrigation method is used, the irrigation control parameters, i.e. intensity and time of water application, can be used to influence surface and subsurface flow. The evaluation of the optimal control parameters corresponds to a search in a two-dimensional space, which is defined by the possible range of the two control parameters. This necessitates either the simulation of scenarios with irrigation models or the carrying out of field experiments. The ensuing assessment serves as the third dimension in the parameter space. The calculation of the optimal parameters thus becomes obvious because it can be compared with finding either the highest peak (or the lowest point) of a mountain range. Up until now, the following methods have been used to find the optimal parameters for the control of water propagation:

- trial and error
- systematic search (also called grid search)
- inverse modelling
- numerical methods of optimization.

**Trail and error**
This is the oldest type of method for controlling water application to the field. This method has to be used in locations where only relatively few calculations (or experiments) can be undertaken. This is particularly the case when running field experiments which necessitate up to several days between each water application in order to re-establish the same initial soil moisture conditions. On the basis of estimated control parameters, the objective function is evaluated for each experiment in order to find an optimal set of control parameters. The trial and error approach is of course combinable with irrigation models.
Irrigation control: towards a new solution of an old problem

Systematic or grid search
In this method, the combinations of the control parameters are systematically varied for a given resolution – somewhere between their minimum and maximum values. The irrigation procedures are simulated (or indeed physically carried out) for all possible combinations and the value of the objective function is calculated. The combination showing the best result is the optimal parameter set for water applications. One major drawback of the systematic search strategy is that it is obviously very time-consuming. However, as long as a sufficiently high resolution has been chosen, this method does allow for the calculation of so-called global-optimal solutions for the parameters.

The purely stochastic search, which principally shares the same characteristics, generates probabilistic parameter combinations with a given resolution within the whole parameter space. Evaluating the objective function for each of these combinations finally provides the best solution. Both approaches are known as blind optimization strategies because the subsequent search steps are fully independent of each other.

Inverse modelling
Inverse modelling can generally be applied if a unique relationship exists between the control parameters and the objective functions. As to micro irrigation with constant water application, the optimization problem can thus be reduced by, for example, considering only the irrigation time as the control parameter and by introducing horizontal and vertical saturation depths as the essential part of the objective function. This then serves to calculate the optimal irrigation time for a desired saturation depth. In the case of numerical process modelling, the inverse solution can only be found via iterative procedures, which requires substantially higher computational effort. Analytical inverse formulae only exist for a few empirical models of micro irrigation.

Numerical methods for nonlinear optimization
The surface/subsurface flow processes involved, e.g. in furrow irrigation, represent highly nonlinear phenomena. The optimal control of such nonlinear processes requires methods of nonlinear optimization if (i) we cannot simplify the optimization problem, (ii) computational effort plays an essential role and (iii) reliable global optimal solutions are required. The application of nonlinear optimization generally employs a direct search by evaluating optimal solutions only on the basis of model results. Gradient-based search procedures are not generally applicable because they also require, in addition to an evaluation of the nonlinear function, the derivatives of the objective function. Unfortunately, their approximation requires considerable effort when using numerical flow models.

The objective function
The objective function, which reflects different irrigational strategies, represents a formal description of all the requirements placed on irrigation control. For the majority of all irrigation problems, the objective function consists of a combination of these requirements, together with several restrictions. The different criteria run-off RO, deep percolation DP,

3 A global optimal solution is the parameter combination with the best values over all other possible parameter combinations. In contrast, locally optimal solutions only provide the best parameter combinations for the direct vicinity.

4 See section 3.3.1.5 or Burt et al. [1997] for the definition of irrigation performance criteria.
Irrigation adequacy $AD'$ and distribution uniformity $DU'$ are generally combined by a weighed sum:

$$\max Z(q,t) = \max \left( w_1 \left( 1 - \frac{RO}{T} \right) + w_2 \left( 1 - \frac{DP}{T} \right) + w_3 AD + w_4 DU \right)$$

(2)

$$(q^*,t^*) = \arg \max Z(q,t).$$

(3)

The weights $w_{1,4}$ need to be subjectively determined in advance by the decision maker – which presupposes a certain experience with the importance and consequences of the objective criteria. Possible solutions are: (i) the use of a pareto-optimal objective function or (ii) a simplification of the objective function via consideration of the relationships between the different criteria (see Table 1). Pareto-optimal objective functions require selecting from a quantity of similar value solutions the one which best corresponds to the subjective preferences of the farmer. The problem of subjective selection has, thus, only changed colour and remains an insurmountable hurdle for the farmer.

Table 1 Objective functions for furrow irrigation depending on the strategy of irrigation planning and the assessment of surface runoff

<table>
<thead>
<tr>
<th>Runoff</th>
<th>Deficit irrigation</th>
<th>Full irrigation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss</td>
<td>$\max Z_z(q,V) = \max \left( AE^V \right)$</td>
<td>$\max Z_z(q,t) = \max \left( w \left( 1 - \frac{RO}{V} \right) + w PAE^V \right)$</td>
</tr>
<tr>
<td>Recycling</td>
<td>$\max Z_z(q,V) = \max \left( AE^V \right)$</td>
<td>$\max Z_z(q,t) = \max \left( PAE^V \right)$</td>
</tr>
</tbody>
</table>

Figure 3 shows the objective function $Z_2$ with a weighing of $w_1 = w_2 = 0.5$. The objective function takes on the shape of a curve instead of that of an expected area because the condition $AD^q = 1$ only allows for a combination of control parameters which moisten the root zone down to a given depth. Especially the projection of the objective function (Figure 4) demonstrates the strong nonlinearity of the objective function, which, in this case, only has one maximum. This unique maximum is only valid for objective functions which are only dependent upon either inflow or irrigation time [Zerihun et al., 2001].

The parameter space for optimal solutions of irrigation control is in general restricted by the following:

i) In the case of demand specific irrigation strategies, the water quantity $V$ must not sink below a certain minimum amount $V_{\min}$ ($V > V_{\min}$).

ii) The inflow $q$ must not exceed a given maximum $q_{\max}$ in order to avoid erosion (furrow irrigation) or surface runoff (micro irrigation) ($q_{\max} > q$).

The objective function exhibits a different form if inverse methods are applicable for evaluating the optimal parameters. In this case, the minimum of the sum of the mean squared error between desired and simulated model results is used. Unfortunately, the existence of local extrema cannot be predicted either.
1.3.1 Modelling of water transport

For satisfying the challenges of the ever growing demands in the world's food production we need to (i) evaluate and optimize existing irrigation systems, (ii) design new, efficient and sustainable systems and (iii) plan the regeneration of hitherto mismanaged systems. Fundamental mathematical modelling can contribute towards meeting these targets because the "description of surface irrigation hydraulics provides a useful, even necessary, framework for mechanisms that can tell quickly, reliably and economically where irrigation water will go under given conditions" [Bassett and Fangmeier, 1980]. Such mechanisms are referred to here as 'irrigation models'.

During the last decades several investigators have proposed a large number of irrigation models of various degrees of sophistication. Some models tend to treat only a portion of the irrigation
process, namely the advance phase, whereas other models describe the complete irrigation process. But, until now, only very limited attempts have been made to develop seasonal irrigation models, which (i) simulate the water flow for alternating irrigation and redistribution times during the entire growing period and (ii) include a prediction of the crop water requirements, the crop growth and the yield forecast in its calculations.

Researchers have placed much emphasis on the description of surface flow hydraulics, whereas infiltration has as yet been rather neglected. Empirical infiltration formulae – mostly the one by Kostiakov-Lewis – are frequently used, but a small number of researchers have recently recognized the importance of more predictive infiltration equations. They try to answer two important questions, namely: *When does the irrigation water reach the plants?* and *How much of the water is lost?* Physically based two-dimensional infiltration models are able to answer these questions more reliably and, thus, may make an important contribution with respect to increasing the efficiency of furrow irrigation systems.

In this section we review various detailed studies of irrigation models to be found in literature. Additionally, we give a brief summary of crop growth modelling approaches. Finally, conclusions are drawn in order to emphasize the need for more research in this area.

### 1.3.1.1 Surface-subsurface flow models

Surface irrigation models are classified according to the classification of surface flow equations, which is described in section 3.3.1.2. The complete hydrodynamic model (HD) employs equations to describe the non-uniform and unsteady surface flow by the conservation of mass and momentum. Different simplifications of the momentum equations lead to the Zero-Inertia model (ZI), the Kinematic-Wave model (KW) and the Volume-Balance (VB) model.

Very often, infiltration is merely treated as a sink term in the continuity equation. The majority of the models only account for the advance phase or for an irrigation event (i.e. as long as surface flow exists) and neglect the water withdrawn by the plants.

Up until now researchers have mainly focused on surface flow hydraulics and the numerical solution of the governing set of equations. A comprehensive review of all irrigation model classes was undertaken in order to get a full picture of the past and present-day developments in furrow irrigation modelling. Table 2 gives a comprehensive picture of furrow irrigation models (together with the irrigation methods to which they can be applied), the solution techniques for the governing equations and the infiltration model which is utilized. These furrow irrigation models and a number of border/basin irrigation models are discussed in detail in Wöhling [2005].

Most irrigation models utilize empirical infiltration equations, either the original Kostiakov equation or the Kostiakov-Lewis equation. Empirical functions are calibrated for specific conditions of the irrigation system and, hence, cannot be assigned to other conditions. These functions are simple intake-over-time relationships, i.e. the infiltrated water volume is only dependent on infiltration opportunity time. However, water infiltrates from irrigated furrows both laterally and vertically, depending not only on opportunity time but also on geometric factors and flow depth. This can hardly be accounted for by these volume-time relationships. Moreover, nothing can be stated about the spatially and temporally varying flow in the root zone which, of course, has a great impact on crop growth.
Table 2  Selected furrow irrigation models

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Model class</th>
<th>Irrigation method</th>
<th>Solution method</th>
<th>Infiltration</th>
<th>Irrigation phases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yu and Singh [1990]</td>
<td>VB</td>
<td>F</td>
<td>implicit iteration</td>
<td>KL</td>
<td>C</td>
</tr>
<tr>
<td>Valiantzas [2000]</td>
<td>VB</td>
<td>F</td>
<td>NR</td>
<td>KL</td>
<td>A</td>
</tr>
<tr>
<td>Mailhol [2004b]</td>
<td>VB</td>
<td>F</td>
<td>Laplace</td>
<td>modified Horton</td>
<td>C</td>
</tr>
<tr>
<td>Eldeiry et al. [2004]</td>
<td>VB</td>
<td>F</td>
<td>NR</td>
<td>KL</td>
<td>A</td>
</tr>
<tr>
<td>Rayej and Wallender [1988]</td>
<td>KW</td>
<td>F</td>
<td>implicit, NR + DS</td>
<td>mKL</td>
<td>C</td>
</tr>
<tr>
<td>Fonteh and Podmore [1994]</td>
<td>KW</td>
<td>F</td>
<td>NR</td>
<td>GA</td>
<td>C</td>
</tr>
<tr>
<td>Raghuvanshi and Wallender [1997b]</td>
<td>KW</td>
<td>F</td>
<td>explicit, iterative fixed-space solution</td>
<td>KL</td>
<td>C</td>
</tr>
<tr>
<td>Elliott and Walker [1982]</td>
<td>ZI</td>
<td>F</td>
<td>DCV, NR +DS</td>
<td>KL</td>
<td>A</td>
</tr>
<tr>
<td>Ross [1986]</td>
<td>ZI</td>
<td>F, Bo</td>
<td></td>
<td>KL</td>
<td>C</td>
</tr>
<tr>
<td>Oweis and Walker [1990]</td>
<td>ZI</td>
<td>F, Bo, Ba</td>
<td>DCV, NR+DS</td>
<td>KL</td>
<td>C</td>
</tr>
<tr>
<td>Strelkoff [1991]</td>
<td>ZI</td>
<td>F, Bo, Ba</td>
<td>NR+DS</td>
<td>KL</td>
<td>C</td>
</tr>
<tr>
<td>Schmitz and Seus [1992]</td>
<td>ZI</td>
<td>F, Bo</td>
<td>analytic ZI + PI</td>
<td>analytic 1D-Ri + TM</td>
<td>A</td>
</tr>
<tr>
<td>Schmitz et al. [2000]</td>
<td>ZI</td>
<td>F</td>
<td>explicit FD, NR</td>
<td>KL</td>
<td>A, S</td>
</tr>
<tr>
<td>Schmitz et al. [2002]</td>
<td>ZI</td>
<td>npF</td>
<td>analytic ZI + PI</td>
<td>none</td>
<td>A</td>
</tr>
<tr>
<td>Walker [SIRMODIII, 2003]</td>
<td>ZI</td>
<td>F, Bo, Ba</td>
<td>NR+DS</td>
<td>KL</td>
<td>C</td>
</tr>
<tr>
<td>Zerihun et al. [2003]</td>
<td>ZI</td>
<td>F, Bo, Ba</td>
<td>implicit FD, NR + DS</td>
<td>1D-Ri</td>
<td>C</td>
</tr>
<tr>
<td>Souza [1981]</td>
<td>HD</td>
<td>F</td>
<td>implicit FE</td>
<td>KL</td>
<td>A</td>
</tr>
<tr>
<td>Schmitz et al. [1985]</td>
<td>HD</td>
<td>F, Bo</td>
<td>implicit MC</td>
<td>1D-quasi-analytic</td>
<td>A</td>
</tr>
<tr>
<td>Wallender and Rayej [1990]</td>
<td>HD</td>
<td>F, Bo</td>
<td>SM</td>
<td>mKL</td>
<td>A, S</td>
</tr>
<tr>
<td>Bautista and Wallender [1992]</td>
<td>HD</td>
<td>F</td>
<td>SM</td>
<td>KL</td>
<td>C</td>
</tr>
<tr>
<td>Tabuada et al. [1995]</td>
<td>HD</td>
<td>F</td>
<td>NR + DS</td>
<td>2D-Ri</td>
<td>C</td>
</tr>
</tbody>
</table>

with: A = advance phase; Ba = basin; Bo = border; C = all irrigation phases; DCV = deformable control volume; DS = double sweep method; F = furrow; FD = finite differences; FE = finite elements; GA = Green & Ampt equation; HD = complete hydrodynamic; KL = Kostiakov-Lewis equation; KW = kinematic wave; MC = method of characteristics; mKL = modified Kostiakov-Lewis equation; NR = Newton-Rawson method; PI = Picard iteration; R = recession phase; Ri = Richards equation; S = storage phase; SM = shooting method; TM = transformation model by Schmitz [1989]; VB = volume balance; ZI = zero-inertia

The combination of physically based infiltration models, like the Richards equation in its one-dimensional or two-dimensional form, and surface hydraulics (ZI) is employed only in rather complex hydrological model packages (e.g. SHE/MIKE SHE by Abbott et al. [1982, 1986], Bathurst et al. [1995], Refsgaard et al. [1995]; Di Giammarco et al. [1994], Morita and Yen [2002]). But more recently, Zerihun et al. [2003] developed a surface irrigation model and coupled the ZI equations with HYDRUS-1D, which utilizes the one-dimensional Richards equation. The Preissmann implicit finite difference scheme is used to transform the ZI equations into a pair of nonlinear equations [Cunge et al., 1980], which are solved by the Newton-Rawson scheme in combination with the double-sweep algorithm. Although Zerihun et al. [2003] turns from empirical infiltration to physically based infiltration, his model still
ignores the two-dimensional character of infiltration from furrows. Crop growth and climatic boundary conditions are not included either. Moreover, the computational effort continues to be high due to the numerical solution method and may be afflicted with stability problems.

Tabuada et al. [1995] have coupled the hydrodynamic flow equations with the two-dimensional Richards equation and adapted the model for furrow irrigation. They use an implicit scheme of finite differences and solve the surface flow equations with the Newton-Rawson method and the double sweep algorithm [Ligget and Cunge, 1975]. An implicit method is used to solve the Richards equation by the Gauss-Seidel method. This irrigation model, however, still features some limitations:

- The discretization of the subsurface flow domain is made in rectangular elements of equal size, which is (i) less flexible for mapping any arbitrary shaped furrow cross sections and (ii) more likely to be afflicted with numerical instabilities as compared to more flexible calculation meshes.
- The irrigation model does not consider the evaporation from the soil surface, the plant-root water uptake or the precipitation.
- The soil-moisture retention curve is described by the model of Brooks and Corey [1966], which fails to offer a satisfactory description of the retention curve in the wet region [Kutilek and Nielsen, 1994].
- The rather complex coupled surface-subsurface model is still afflicted with the disadvantages known for numerical solutions (as mentioned above). Additionally, it requires a long computational time which is reported to limit practical application.

1.3.1.2 Crop models

There have been various recent efforts on process-based crop simulation (e.g. Brisson et al. [1998], Tsvetsinskaya et al. [2001], Nagai [2002]) accompanied by a detailed early mechanistic model review by Whisler et al. [1986]. In the context of surface irrigation, crop growth is usually described at the plant scale. Microscopic studies – i.e. models on the organ scale, cell scale, etc. – are not effective in irrigation modelling [Mathur and Rao, 1999]. Additionally, the available model input is often limited to only daily climatic values (precipitation,
temperature, solar radiation, etc.) and the microscopic models require input data with a much higher resolution.

Detailed studies of plant composition and transpiration lead to the conclusion that about 95% or even more of the water extracted from the soil by the plant roots is transpired and flows through the soil-root-stem-leaves-atmosphere system [Kutilek and Nielsen, 1994]. The resulting concept of a soil-plant-atmosphere continuum describes the flux through the above system with an analogy to current in an electrical resistance network (cf. e.g. Oke [1987], Monteith and Unsworth [1990], Larcher [1994], Kutilek and Nielsen [1994]). An extensive review on models using the so-called macroscopic approach can be found in Mathur and Rao [1999].

A suitable simplification to quantify the potential water uptake by the plant roots is the assumption that the water uptake equals the potential transpiration. Although there is time shift or diurnal variation between the two processes, this is a reasonable assumption for daily averages. In irrigated agriculture, the potential crop evapotranspiration, i.e. the composite of potential transpiration and potential evaporation, (often referred to as crop water requirements) is usually calculated by (i) estimating the reference evapotranspiration, $ET_0$ and (ii) multiplying $ET_0$ by a crop coefficient, $K_c$, as described by Doorenbos and Pruitt [1992] and Allen et al. [1998].

1.3.1.3 Irrigation model review – conclusions

In contrast to volume balance and kinematic wave models, hydrodynamic models (HD) provide a realistic description of the surface flow for a large range of a priori unknown conditions. On the other hand, HD models are complex and laborious, require much computational effort and the majority of the reported solution techniques are afflicted with numerical problems. An acceptable compromise are zero-inertia models, which are nearly as accurate as HD models [Esfandiari, 1997] but numerical problems are less likely. Moreover, an analytical solution of the ZI equations exists [Schmitz and Seus, 1992], which accounts for any chosen infiltration function.

Attempts to couple the surface flow equations and physically based two-dimensional infiltration were very limited in the past (cf. Table 2) and do not fulfil the requirements for a
seasonal irrigation model. The entire growing season with crop growth simulation is only incorporated in some of the rather conceptional/empirical irrigation models. Not one of the irrigation models combines an accurate but robust and numerically efficient surface flow model with a process-based two-dimensional infiltration model which includes soil water transport, root water extraction, evaporation and precipitation.

1.3.2 Optimization irrigation control

1.3.2.1 Current optimization methods in micro irrigation

With the help of a specific controlled infiltration, micro irrigation has the potential to provide plants with an irrigation efficiency of around 95%. Thanks to highly precise drippers, the prescribed infiltration rate recommendations can be adhered to almost exactly. Nonetheless, there is a need for optimal control with respect to the water movement in the soil. This is characterized by Dasberg and Or [1999]:

*Present-day drip irrigation design practices tend to emphasize system hydraulic performance (pressure distribution, filtration, emitter uniformity, etc. – all of which may be optimized with the aid of computers), while agronomic-hydrologic considerations, such as emitter-soil-plant interactions, receive less attention or are dealt with empirically.*

The optimal control of soil water transport generally employs the inverse methodology. In this case, the known vertical and horizontal space of the root zone has to be supplied with water. This allows computing the necessary inflow rate and the time of water application. For this task, irrigation practice mostly uses empirical models (e.g. Zur [1996]) which allow a simple evaluation of the control parameters. This advantage is however questionable when considering the consequences of significant simplifications with respect to the description of soil water transport. Moreover, the empirical character of the formula only covers a relatively small number of soil types. The inverse method proposed by Bresler [1978] represents more or less the guidelines for problem solving with respect to analytical models (e.g. Revol et al. [1997]) and also for empirical models [Healy and Warrick, 1988]. This, however, necessitates prescribing both the infiltration rate and the lower limit of the soil moisture corresponding to a certain matrix potential.

Up to now it would seem that only Schütze et al. [2005b] have realized the use of numerical flow modelling for the inverse type of irrigation control. The computation of the optimal irrigation rate is performed by minimizing the sum of the least squared error between the prescribed and calculated extension of the wetted soil volume. When using the simplex algorithm according to Nelder and Mead [1965], the contribution of Schütze et al. [2005b] required a relatively large effort as regards the determination of the parameters. This is due to a low convergence rate of the simplex algorithm, which required on average more than 10 simulations of the numerical subsurface flow model Hydrus-2D.

Considering the larger area of these types of micro irrigation problems, it can be stated that in irrigation practice, only empirical methods are applied. The other proposed approaches on the basis of flow modelling are neither simple to apply, nor do they allow for a rapid computation of the optimal control parameter.
1.3.2.2 Current optimization procedures in furrow irrigation

The optimal control of furrow irrigation focuses on the modelling and controlling of surface flow processes, whereas the optimal soil/water distribution only plays a minor role. Literature basically only presents two approaches for controlling the on-field water distribution. The first type of approach deals with the real-time control of the inflow, which aims towards a uniform distribution of the irrigation water along the furrow (e.g. Camacho et al. [1997]; Turnell et al. [1997]). This is achieved by using pressure sensors along the furrow in order to register the wave propagation. On the basis of the planned wave propagation, a surface flow model computes a new inflow rate, which will subsequently be realized at the furrow entrance. This procedure requires an extreme technical and financial effort and, thus, does not deserve more than a brief mention in this contribution.

The subsequently described second type of optimization approach is widely used for determining the optimal control parameters on the basis of simulation. The currently used model systems are somewhat similar and consist of (i) zero inertia (SRFR or SIRMOD) and kinematic wave or volume balance models for modelling surface flow, (ii) the Kostiakov approach for the portrayal of the infiltration and (iii) water balance models in order to describe the soil water balance. With these models, the soil water distribution cannot be quantified and it is supposed that the water serves exclusively the root zone of the plants until the field capacity of the soil is exceeded.

A complete implementation of the methodology (ii) has been achieved by the Furrow Irrigation Design Optimizer (FIDO) by McClymont [2000]. He maximizes the objective function (Equation 2) by using the simplex algorithm in order to determine both the optimal flow and time of irrigation. Besides the subjectivity when weighing the different components of the objective function, the deficiencies of this approach also lie in (a) a significant simplification in modelling the soil water flow, (b) long computation times due to the slowly converging simplex algorithm and (c) the fact that some components of the objective functions, e.g. percolation, can only be approximately estimated.

The approach of McClymont [2000] is roughly followed up by Raghuwanshi and Wallender [1997a], Kiwan [1996] and Zerihun et al. [2001] with some small deviations. Raghuwanshi and Wallender [1997a] applied a systematic search which determined the values of a reduced objective function for 21 inflows and 3 degrees of irrigation efficiency. The inflow corresponding to the maximum value of the objective function is then considered as optimal. The well-known procedure of Walker and Skogerboe [1987] uses water balancing for portraying surface flow and likewise evaluates the optimal inflow on the basis of a systematic search.

In contrast to these approaches, and in order to solve the optimization problem as far as possible in an analytical way, Kiwan [1996] and Zerihun et al. [2001] derive regression relationships between control parameters and optimal criteria from the hydraulic model. Notwithstanding this fact, they finally have to employ iterative procedures for calculating the optimal control parameters.

On the basis of simple trial and error approaches for achieving an optimal irrigation control for cotton, Smith et al. [2005] achieve an improved water application efficiency of more than 60% compared with a starting value of 44%. They show that this value can be even further improved up to 90% by using nonlinear optimization procedures. In this context they demonstrated that, in general, the traditional farming methods lead to over-irrigation, i.e. a considerable percentage of the water loss occurs by percolation into the groundwater.
The control of furrow irrigation is often confused with the operational planning of irrigation. In order to simplify the originally coupled, nonlinear optimization problem consisting of control and scheduling, Reddy and Clyma [1981] and Holzapfel et al. [1986] assume a fixed irrigation frequency, e.g. 10 days. This is justified by regional water legislation. They optimize inflow, furrow length and irrigation time on the basis of an objective function which aims to maximize net profit. For solving the optimization problem, both methods rely on different types of linear programming and, thus, they have to introduce numerous simplifications. These include, amongst others, the linearization of the modelling system as well as neglecting to assess percolation and surface runoff.

1.3.2.3 Summary of the current problems in irrigation control

With respect to irrigation control, current practice does not at all exploit the potential contained in the prognostic process modelling of surface-subsurface flow processes. As to micro irrigation, model application is in practice restricted to the use of empirical models, whereas furrow irrigation currently employs dynamic models of surface flow, combined with empirical infiltration formulae for controlling water application. A few studies combine nonlinear optimization with dynamic flow models; however, these are seriously restricted due to the growing computational effort, as well as the complexity and instability of the numerical methods.

1.3.3 Optimizing irrigation scheduling

In literature we can only find a few approaches with respect to the global optimization of irrigation scheduling. These efforts are generally limited to identifying irrigation schedules on the basis of a sensor-controlled irrigation strategy using water balance modelling as for, for example, CROPWAT [Smith, 1992]. On the basis of such approaches, the relevant studies demonstrate that a successful irrigation control is possible with respect to the scheduling of a simple deficit irrigation strategy. However, because the efficiency of single irrigation cycles can only be estimated, their respective inter-dependency with the irrigation scheduling is not able to be fully taken into account.

With respect to this furrow irrigation problem, Raghuwanshi and Wallender [1997b] present an approach based on fixed intervals between the irrigation cycles for solving this problem with respect to furrow irrigation. Bearing minimal water consumption in mind, both the optimal inflow and the irrigation time are evaluated using a grid search for each single irrigation. The investigation, which uses water balance modelling, does not directly combine the Kostiakov-based infiltration computation with irrigation advance. This seriously affects the reliability of the simulation. The irrigation stops when the infiltrated water attains 80% of the adequacy (the portion of the infiltrated water volume with respect to the total volume necessary for filling up the usable soil water storage). Unfortunately, this approach does not allow achieving a global optimal irrigation scheduling due to the subjective fixing of the irrigation times and because of failing to pay sufficient attention to the inter-dependency between the single irrigation events.

Bras and Cordova [1981] introduce a global approach for optimal irrigation scheduling on the basis of dynamic programming. In order to maximize the harvest, they propose an optimal distribution of a given total amount of water for a single irrigation cycle. However, dynamic programming necessitated a substantial reduction in the complexity of the problem. The
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Investigation was therefore based on a regular interval between the single irrigation events, and the discretization of the used water balance model was relatively coarse. Introducing a stochastic state transition helps to estimate the error caused by the coarse discretization and it also allows the implicit use of a simple stochastic precipitation model.

A further measure for reducing the computational effort consists in selecting an extremely simple harvest model, which, however, fails to take into account the interconnection between the different growth periods and the varying stress sensitivity of the plants. Such simple harvest modelling is criticized by Rao et al. [1988] who proposed, as an alternative, a two-step optimization in order to limit the computational effort arising from the use of a more complex harvest model. The first step divides the whole growth period into four phases of different stress sensitivity. The available water is then optimally distributed to the four phases by dynamic programming. Also on the basis of dynamic programming, the water in each of these four phases is optimally assigned to the single irrigation events in a second optimization step, whereby fixed intervals between the single irrigation events are assumed. A further development of the approach proposed by Sunantara and Ramirez [1997] avoids separating the optimization process. Moreover, daily irrigation decisions allow optimizing the amount and date of the different irrigation events. In addition, the search space is extended by a component for memorizing the time and date of the last precipitation – this allows the explicit use of a stochastic climate model. Besides the optimal scheduling and computation of the harvest as a function of the total water used, evaluations are also done for optimal schedules and harvests involving water quantities that are less than the total amount available. This allows a highly efficient approximation of the crop yield function.

The main disadvantage of the optimization strategies proposed by Bras and Cordova [1981], Rao et al. [1988] and Sunantara and Ramirez [1997] lies in the necessary discretization of the water transport models and, more specifically, of the soil moisture content. This seriously limits the predictive reliability of the models, which, in turn, affects the schedules and, thus, causes substantial problems as regards the optimal control of single irrigation events. Optimal irrigation control requires a reliable simulation of the surface flow, as well as a rigorous portrayal of the soil conditions. However, due to the growing complexity, this cannot be achieved by dynamic programming. Although the process modelling of soil water transport offers a far more accurate portrayal of reality, it is unfortunate that all the scheduling procedures proposed up until now in literature rely solely upon water balance models.

1.3.4 Current irrigation practice: deficits

The preceding overview and critical analysis of current practice in irrigation control and scheduling revealed significant deficiencies which seem to obstruct the way to a highly efficient and sustainable irrigation management. In order to overcome these hurdles, the following points need to be satisfied:

- Physically based irrigation models are the most important prerequisite for a sustainable irrigation management policy. Possible future irrigation scenarios which may not yet have been experienced can only be generated with the help of such models.
- Simulation-based scheduling methods can answer the farmers and policy makers' 'what if' questions more quickly and efficiently than on-farm trials can.
- An optimization of the control and scheduling parameters is the basis for determining the best irrigation management strategy.
• Especially in the case of deficit irrigation, the irrigation scheduling has an influencing effect on irrigation control (and vice versa). Because of this mutual, two-way interaction, the optimization of both tasks must be carried out simultaneously.

• In addition to experimentation, the effect of different scheduling strategies can be effectively studied through modelling, with the use of actual or stochastically produced long-term climate data.

• From a societal perspective, optimum irrigation may be defined more broadly as the maximization of overall benefits, including such non-monetary benefits as water quality protection, food security, increased employment and the resettlement of populations.
The optimization of irrigation management for a more efficient and sustainable irrigation agriculture is a very complex task. The problem can only be efficiently solved with the help of two complimentary approaches: physically based process modelling backed up by innovative optimization strategies based on artificial intelligence and genetic algorithms. If classical optimization methods are used, two problems arise. On the one hand, the computation time required for the best solutions is unrealistically long and, on the other hand, the numerical simulation and optimization is far too complicated for the average, untrained user (i.e. the farmer).

As outlined subsequently, novel approaches in the domain of artificial intelligence can solve this dilemma by designing procedures which are simple and robust to use, yet rapid in their calculations. These innovative procedures have one slight inherent drawback in that, after training, the resulting tool is restricted to applications at the site which provided the regional data for setting up the GAIN-P methodology and, thus, must be set up anew for other irrigation systems with significantly different soil and environmental data. This, however, is not a serious disadvantage for two reasons; firstly, GAIN-P needs only to be set up once – and once only – for a specific area and, secondly, training can be routinely performed at irrigation research centres, or universities, and does not need to be carried out on site.

The innovative GAIN-P strategy thus combines two procedures resulting from research in artificial intelligence: artificial neural networks (ANN) and genetic, or evolutionary, algorithms (GA or EA), used in combination with rigorous process modelling.

With the optimization of irrigation management in mind, the following chapters present the essential principles with respect to the interaction between artificial intelligence and novel process modelling methods.

2.1 Using neural networks for transferring complex physically based irrigation models into a simple tool for irrigation control

In those areas which have a long tradition of irrigation agriculture, the irrigation methods which have been handed down through the ages actually manage to achieve surprisingly good
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results. The knowledge which is passed down from father to son is based on years of experience and the observation of many thousands of irrigation events. This acquired expertise is supported by an intuitive feeling with respect to the weather and its implications for the harvest and enables favourable, intuitively based, decisions which result in the positive adjustment of the irrigation schedule to suit the particular weather conditions. This type of optimization of the irrigation parameters corresponds, technically speaking, to the farmer's trial and error approach with his associative memory capacities. Its success relies on the experience gained from an immense number of applications during one growing season, and this repeated over many years. The farmer memorizes the ensuing crop yield which results from a certain irrigation strategy and then remembers this in combination with the prevalent weather conditions of that particular season. For several decades now, irrigation experts have been trying to make up for their lack of comprehensive empirical knowledge by using optimization algorithms in combination with mathematical models. These models are intended for use in new areas which are to be placed under irrigation or where already irrigated areas are to be restructured or expanded. However, the aforementioned deficiencies in classical modelling technologies necessitate a critical rethinking of current modelling and optimization philosophy. In this context, the traditional trial and error approach combined with an associative memory substantially contributes towards finding the overall solution to the problem. The carrying out of numerous real-time irrigation procedures is substituted by the simulation of irrigation scenarios with physically based flow models. This involves replacing the comprehensive empirical knowledge gained by long periods of observation by a deeper understanding of the complex processes involved and by positively exploiting local observation data and, for example, soil parameters such as the soil hydraulic properties. However, due to the insurmountable computational effort required with respect to process modelling, this type of methodology can only lead to an ideal solution when combined with an artificial neural network. Thus, for determining the suitable control parameters for an irrigation event, two main actors come into play. One of the leading roles is occupied by physically based flow models, which replace the comprehensive empirical knowledge with a realistic portrayal of, and insight into, the complex processes involved. The second actor takes on the role of an artificial neural network and is responsible for accumulating the comprehensive empirical knowledge with respect to finding the most suitable combination of irrigation parameters to be used. Such a use of artificial intelligence implicitly provides the possibility for a simple and straightforward application of sophisticated process models. In terms of an irrigation event, this enables an efficient and sound determination of the parameters for the water application. Instead of using process models to assist with the problematic direct optimization of an objective function, we use a procedure which considers the flow modelling and the determination of the irrigation parameters as two separate, distinct steps. During the first phase, after being fed with local data, the process models generate a comprehensive range of irrigation scenarios (Figure 7). With the help of the thus obtained parameters and the input/output data sets, the neural network learns the behaviour patterns of the irrigation system under consideration.

During the second step, the artificial neural network states the optimal irrigation parameters for the field, i.e. the one which had featured in the training phase. It does this by analyzing the local field specific data and by calculating the parameters which can produce, for each stage of the growing phase, the most favourable soil moisture profile for the crop. In so doing, the plant's requirements during each of the different growth phases throughout the season are completely catered for.

The complex physically based irrigation models are difficult and cumbersome to operate due to their numerical nature. If they are substituted by straightforward, robust and simple-to-
operate artificial neural networks purely for simulating surface and subsurface flow, then this is called straightforward modelling. This is in contrast to inverse modelling, where the artificial neural network is used to solve the inverse problem, i.e. to determine from a desired soil moisture the corresponding water application parameters. For straightforward modelling, the neural network must learn the following transformation $f$:

$$f : (\vec{x}_{\text{ini}}, \vec{x}_{\text{bound}}, t) \rightarrow \vec{y}$$

along with the initial conditions $\vec{x}_{\text{ini}}$, the defined boundary conditions $\vec{x}_{\text{bound}}$ and $t$ for the irrigation time. During the training phase for this application (Figure 7), the neural network functions parallel to the irrigation model and calculates a particular result $\vec{y}'$ for each training step, which is then compared with the irrigation models result $\vec{y}$. Throughout the training phase, the learning procedure aims at minimizing the approximate error $\| \vec{y} - \vec{y}' \|$ by changing the weights of the neural network.

When using the irrigation model for generating the training vectors $(\vec{x}, \vec{y})$, this can be done either simultaneously (online, see Figure 7) or beforehand (offline, see Figure 7). With offline

![Diagram](image-url)
learning, the simulated scenarios $D$ are first of all stored in a training data bank. The training data set is achieved by randomly or systematically varying the elements of the input data set $(\tilde{x}_{\text{ini}}, \tilde{x}_{\text{bound}}, t)$. The ranges of the individual elements will finally determine the scope of validity of the trained neural network. The cardinality of the training set $n_D$ finally dictates the quality of the approximation of the mapping $f : (\tilde{x}) \rightarrow \tilde{y}$, i.e. it stipulates the quality of the neural network's simulation of the irrigation processes. All the other parameters of the irrigation model, i.e. the field length, the roughness of the furrows or the soil hydraulic characteristics, are known as internal parameters. They are given parameters and are for this reason constant. After training, the ANN can be applied in different mapping directions.

For simulating the subsurface flow the ANN then provides the mapping $f$:

$$f : (\tilde{x}_{\text{ini}}, \tilde{x}_{\text{bound}}, t) \rightarrow \tilde{y}$$

with initial conditions $\tilde{x}_{\text{ini}}$, the simulation time $t$ and boundary conditions $\tilde{x}_{\text{bound}}$. The solution of the inverse problem by a neural network is defined by the following mappings $f_1', f_2'$ and $f_3'$:

$$f_1' : (\tilde{y}, \tilde{x}_{\text{ini}}, t) \rightarrow \tilde{x}_{\text{bound}},$$
$$f_2' : (\tilde{y}, \tilde{x}_{\text{bound}}, t) \rightarrow \tilde{x}_{\text{ini}},$$
$$f_3' : (\tilde{y}, \tilde{x}_{\text{ini}}, \tilde{x}_{\text{bound}}) \rightarrow t.$$

Depending on the selected mapping task, the external parameters – namely the simulation time $t$, the initial conditions $\tilde{x}_{\text{ini}}$ and the boundary conditions $\tilde{x}_{\text{bound}}$ – serve as input or output parameters during the application of the trained ANN. These parameters and the simulation results $\tilde{y}$ can be varied within the bounds of the generated database when applying any of the four different mapping functions. While the mapping function $f$ can be used as a simulation model, the defined inverse functions are applicable to control problems (e.g. $f_1'$ and $f_3'$ in irrigation), to monitoring tasks (e.g. $f_1'$ and $f_2'$) or likewise to water resources planning (e.g. $f_1'$ and $f_3'$).

### 2.2 Solving complex optimization problems in irrigation with evolutionary algorithms

Using complex simulation models for the purpose of optimization is a very complicated process which cannot be solved with standard conventional optimization procedures such as linear programming or gradient-based methods. Conventional procedures are sufficient for finding optimal solutions with respect to relatively simple tasks such as finding a local maximum or minimum, where it then represents the so-called global optimal solution. On the other hand, when it comes to finding optimizations with the help of complex simulation models, the procedure is laden with complicated problems. Trying to determine the best parameters for the optimal scheduling of a growth period is a typical example of this type of problem. The following factors can all cause difficulties:

- The optimization problem, i.e. the search space, is highly dimensional.
The GAIN-P strategy

• The calculation of the values of the objective function (here, the simulation of a growth period) is relatively complex and time-consuming.
• The objective function is multi-modal, i.e. it has a multitude of local optimal solutions.
• A high computational effort is necessary for calculating the derivations of the objective function.

For solving these types of optimization problems, conventional optimization procedures require either an inordinate amount of time, or they are not appropriate for solving the problem in question. In such cases, only approximative optimization procedures can successfully perform the task. Despite the fact that these are no longer in a position to find the theoretically best solution, they can identify values of the objective function which closely approximate the absolute optimum and, furthermore, the time required for their calculations is incomparably shorter than when applying classical methods.

Evolutionary algorithms (EA) represent an alternative to classical optimization methods when dealing with objective functions which feature many local minima. They imitate genetic principles found in nature. An EA begins its search with a random set of solutions called 'population' which in our case is a random set of schedules. Every solution is assigned a fitness value which is directly related to the objective function – in the case of the scheduling optimization problem this is, for example, the achieved crop yield of a certain simulated irrigation scenario. Thereafter, the population of schedules is modified to a new population by applying four steps similar to natural genetic operators – selection, crossover, mutation and reconstruction. Here is a brief summary of the way in which an evolutionary algorithm functions (for details, see section 3.2):

Step 1 (selection)
In the selection phase, a number of schedules are chosen randomly from the population and the best schedule within this group is selected as parent. This process is to be repeated for as long as individuals are to be chosen, i.e. according to the number of schedules in a population.

Step 2 (crossover)
Crossover produces new schedules by combining the information contained in two parents chosen in the selection. This is done by combining together the single water applications of two randomly chosen parent schedules in order to form a new schedule.

Step 3 (mutation)
By mutation, the variables of each water application, i.e. when and how much to irrigate, are randomly altered. These variations (mutation steps) are generally small. They are to be applied to the variables of each water application in each new schedule.

Step 4 (reconstruction)
Schedules of the new population are reorganized in order to match the constraints of the given optimization problem.

After the prescribed steps are applied to the whole population, one generation of an EA is completed. The algorithm iterates until a certain desired degree of accuracy is reached. EAs usually necessitate some function evaluations for convergence which unfortunately requires some computational effort because they do not use the derivative information of the objective function. This, however, is compensated for by our problem-adapted EA; it combines the flexibility of restricting the parameter space for valid solutions with the additional possibility of an extensive parallel processing.
2.3 The coupled optimization problem in deficit irrigation

With deficit irrigation, the plants are consciously under-supplied with water and a reduced crop yield is accepted as the penalty. Different crops react differently to water stress and therefore, in respect of the consequences for the harvest, the effects of the water stress is very crop-specific. Potatoes, for example, are not at all suitable for deficit irrigation because they react very negatively to water stress. On the other hand, corn reacts positively and the penalty in harvest terms is slight compared to the generous benefits in water-saving. What is more important to note, however, is that each plant's level of water-stress sensitivity fluctuates with respect to its different growth phases. For this reason, when laying down the irrigation schedules for an entire growth period, it is important to decide beforehand when the growth phases requiring generous irrigation water volumes will occur and, on the other hand, when smaller volumes will suffice. The success of an irrigation plan depends on:

- an appropriate, crop requirement-oriented dividing up of the season's entire water irrigation volume between the different irrigation events
- a spatially homogeneous distribution of the prescribed water volume over the field, in an attempt to improve the overall irrigation efficiency.

As far as deficit irrigation is concerned, both factors can be controlled by adjusting the flow rate. This is because only a limited amount of water is available for each irrigation and, thus, the rate of flow dictates the duration of the event. Both factors, i.e. the rate of the optimum flow and the amount of irrigation water to be used (which is prescribed by the irrigation plan), depend on the initial amount of the averaged soil moisture content in the field prior to irrigating. This, again, obviously strongly depends on the amount of time which has passed since the last irrigation event and, thus, is a direct function of the irrigation-scheduling strategy.

On the other hand, the chosen rate of flow has an inevitable influence on the operative irrigation planning. If, for example, runoff is to be avoided, then the ideal irrigation plan should consist of many short irrigations comprising a small volume of water, yet with a constant high flow. With such a strategy, an improved irrigation efficiency can only be achieved if the irrigation is stopped the moment the quick-flowing water reaches the end of the field. Alternatively, if a larger volume of irrigation water is available, a higher irrigation efficiency can only be attained by decreasing the flow rate. For such a case, the ideal irrigation plan would prescribe fewer irrigation events (with a decreased flow rate, yet with a larger volume of water over a longer period of time). A reduced number of irrigation events throughout a growing period can, however, be disadvantageous for crop development. This is because the stages in the plant's growing phase where increased water is required, i.e. the periods where the plant is the most (negatively) sensitive to water stress, can only be roughly estimated and, thus, not properly catered for.

Optimizing both the control and schedule parameters in furrow irrigation is treated as a nested problem (Figure 8): (i) optimizing the control parameters for each single water application, which is referred to as the 'inner optimization' and (ii) optimizing the irrigation schedule (i.e. the number and date of applications) over the whole growing season, which is referred to as the 'outer optimization'. The objective of the global (nested) optimization is to achieve maximum crop yield with a given, but limited, water volume, which can be arbitrarily distributed over a number of irrigations. The impact of different irrigation schedules on the crop yield is calculated by the furrow irrigation model (Figure 8).
The given global optimization problem is then:

\[
\max Y(S) : S = \{ (t_1, V_1, q_1), \ldots, (t_n, V_n, q_n) \} \quad n, t_i \in \mathbb{N}; q_i \in \mathbb{R}
\]  

(7)

with the optimal solution for maximizing the yield \( Y \):

\[
S^* = \arg \max Y(S)
\]  

(8)

where \( S^* \) is the optimal schedule for a whole growing period consisting of \( n \) irrigations which are defined by the date \( t \), the irrigation depth \( V \) and the inflow rate \( q \). Now suppose that this problem is partitioned into schedule and irrigation-control sub-problems by partitioning the vector of the system parameters \( s \) into two vectors, \( s_s \) and \( s_c \) of the schedule and irrigation control variables:

\[
S = \{ s \}_n^* \quad s_s = \{ (s_{i,j}, s_{i,j}) \}_{j=1..n} \quad s_{c,j} = (t_i, V_i), \quad s_{c,j} = q_i.
\]  

(9)

Nested optimization subsequently solves two optimization problems: the outer optimization problem:

\[
\{ s \}_n^* = \arg \max Y(\{ (s_{i,j}, s_{i,j}) \}_{j=1..n}) \quad (10)
\]

and a number of \( n \) inner optimization problems:

\[
s_{c,j}^* = \arg \max I_{last}(s_{i,j}^*, s_{c,j}) \quad i = 1 \ldots n.
\]  

(11)
The outer loop optimizes the yield $Y$ of the plant with respect to the $n$ irrigations, with control parameters optimized for each of the irrigation events. The inner optimization finds the optimal flow rate $q$ with respect to a high cumulative infiltration $I_{last}$ at the end of the furrow for the $i^{th}$ irrigation event with optimal date $t_i$ and irrigation depth $V_i$ generated by the outer optimization loop. The latter objective implies a high distribution uniformity and low runoff for each irrigation.

The outer optimization has the specific feature that the number of optimization variables, i.e. the number of irrigation events, is a priori unknown. Three constraints are subsequently established in order to determine a set of feasible schedules. The application in deficit irrigation requires

$$\sum_{i=1}^{n} V_i \leq V_0$$  \hspace{1cm} (12)

i.e. the sum of the irrigation depth for each water application must not exceed a given water volume $V_0$. Secondly, the time between two irrigations $s_i$ and $s_j$ may not fall below a minimal value $t_{\text{min}}$ in order to exclude schedules with an unusually high frequency of water applications which tend to result in high costs for labour and the maintenance of the irrigation system. On the same basis as above, each irrigation depth is restricted to a minimal irrigation depth $V_{\text{min}}$ per water application:

$$V_i \geq V_{\text{min}}.$$  \hspace{1cm} (14)

The solution of the $i^{th}$ inner optimization problem depends on the mean initial field water content $\theta_i$ and the given irrigation depth $V_i^*$:

$$q_i^* = \arg \max I_{last}(V_i^*, \theta_i).$$  \hspace{1cm} (15)

The calculated optimal flow rate determines the cut-off time of the irrigation event and is restricted by the constraint in Equation 14.

### 2.4 GAIN-P at a glance

It is difficult to solve the global optimization problem because the target function has many locally optimal solutions and features an undefined number of optimization variables because the number of irrigations is a priori unknown. Thus, finding the global solution is not possible with classical deterministic optimization techniques. For this reason, a new methodology GAIN-P combines Genetic (evolutionary) Algorithms, Artificial INtelligence techniques and rigorous Process modelling for substantially improving irrigation efficiency. A made-to-measure evolutionary optimization technique (see section 3.2) is employed to find a near-optimal solution of the outer optimization problem (when and how much to irrigate) within acceptable computation time.
For efficiently solving the inner optimization problem, namely, the determination of the control parameters for each water application (inflow and cut-off time), a problem-adapted artificial neural network (ANN) based on self-organizing maps (SOM) was developed (see section 3.1). Already after one single training the new architecture allows performing simulation tasks as well as solving inverse problems: the Self-Organizing Map with Multiple Input/Output option (SOM-MIO). The SOM-MIO portrays the inverse solution of the coupled numerical surface/subsurface flow model and, thus, enormously speeds up the overall performance of the complete optimization tool.

For training the SOM-MIO with realistic scenarios, we apply the rigorous and physically sound, coupled surface-subsurface flow model (see section 3.3). The model is based on an analytical zero-inertia surface flow model iteratively coupled with the numerical code HYDRUS-2D, which simulates subsurface flow by the modified Richards equation. In addition to the obtained accuracy, this also permits the consideration of soil evaporation and precipitation as well as root water uptake by plants.
3 Methods: the modules of GAIN-P

3.1 Artificial neural networks (ANN)

In 1986 Rumelhart and McClelland developed the backpropagation learning algorithm for multilayer perceptron networks (MLPs) and, in so doing, paved the way for widespread ANN applications. In fact, ANN represent a different paradigm for computing (see Table 3).

<table>
<thead>
<tr>
<th>ANN - Principle</th>
<th>von Neumann - Computer architecture</th>
</tr>
</thead>
<tbody>
<tr>
<td>memory/processing: no distinct separation ↔ separated processing and memory units</td>
<td></td>
</tr>
<tr>
<td>parallel processing with simple processing units ↔ sequential processing with sophisticated processing units</td>
<td></td>
</tr>
<tr>
<td>programming not necessary – needs training ↔ data processing by an explicit program</td>
<td></td>
</tr>
<tr>
<td>high degree of interconnections – slow transfer ↔ single bus interconnection between CPU and RAM – fast transfer</td>
<td></td>
</tr>
<tr>
<td>Training: change the weight proportional to the difference between desired and actual output ↔ Programming: write step by step what the computer should do</td>
<td></td>
</tr>
</tbody>
</table>

The outstanding results achieved by using MLPs in language and communication, in the control of nonlinear processes and for the approximation of functional relationships created a wave of euphoria which reached the domain of hydrosciences around the beginning of the 1990s (see section 3.1.1). This network architecture is still the preferred method in many sectors and features heavily in research, especially where hydrology is concerned. Just as is the case with the so-called parameter free statistical models, MLPs do not suffer from any restrictions with respect to certain modelling assumptions. It is no longer necessary (!) to ask whether a chosen category of model is at all suitable for the solving of a particular problem. So, in response to
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the question, *Where can artificial neural networks be of assistance?*, possible answers might be:

- where we cannot formulate an algorithmic solution
- where we can get many examples of the behaviour we require
- where we are able to pick out the structure from existing data
- where we need to approximate complex nonlinear models
- where we want to identify multidimensional functions or the parameters of a system.

The structure of the neural network combined with the backpropagation learning algorithm is, in principle, so flexible that no end of arbitrary functional relationships can be described with one sole network [Kolmogorov, 1957]. For this reason, Minns and Hall [1996] referred to MLPs as the 'ultimate black box'; nonetheless, at the same time, they pointed out the essential basic drawback in their application: *The ANN is the prisoner of its training data*. This makes it abundantly clear that the quality and success of the MLPs training is directly dependent upon the quality, relevance, amount and range of the training input data.

As regards the application, two different principles lead to different fields of application in hydrology:

i) Direct (naive) application to data

- learning from observed rainfall-runoff time series – subsequent application of ANN to flood forecasting, which, due to the limited range of observations, remains rather unreliable
- nonlinear regression analysis, e.g. for identifying the relationships between soil parameters and the results of laboratory experiments.

ii) Indirect application to simulation data via mathematical modelling

- For **current types of ANN**: learning from the simulations of comprehensive irrigation scenarios via an irrigation model – applications of ANN simply for performing straightforward simulation tasks
- For **current types of ANN**: learning from the simulations of comprehensive irrigation scenarios via an irrigation model – changing the input into output vector and vice versa for evaluating water application parameters – applications of ANN exclusively for solving the inverse problem
- For the **new SOM-MIO type of ANN**: learning from the simulations of comprehensive irrigation scenarios via an irrigation model – direct applications of ANN for performing either the simulation task or for solving the inverse problem.

However, the availability of suitable training data does not necessarily guarantee convergence of the training algorithm. Once the most important basic principles behind the multilayer perceptron networks and behind the backpropagation algorithm have been discussed, we will deal with this second essential basic shortcoming.

3.1.1 The application of standard neural networks in hydrology

It is more than obvious that in the future MLP networks will play a major role in researching answers to hydrological problems. This is confirmed by the results of a 10-year development phase which was documented up to 2001 in the survey articles of ASCE Task Committee on Application of Artificial Neural Networks in Hydrology [2000a], Dawson and Wilby [2001]...
and Maier and Dandy [2000]. Approximately 90% of the more than 100 commented articles use MLPs together with the standard backpropagation learning algorithm or its modifications.

It is thus apparent that for all fields of application, MLPs have proven themselves over other types of network architectures. It is even the case that MLPs are now exclusively used for rainfall-runoff modelling, for the prediction of river water levels and/or for the modelling of water quality. In general, multilayer perceptron networks achieve similar or even better results even when compared with conventional mathematical models, e.g. conceptual models or regression methods. Where additional pragmatic criteria are to be considered when judging the efficiency of other alternative models, MLPs, once training is completed, show themselves to be incomparably straightforward, robust and rapid in terms of application. What is more, the application of conventional models requires at least some knowledge of hydrology as well as a familiarity with simplifying assumptions and this is not the case when dealing with MLPs. The ASCE Task Committee on Application of Artificial Neural Networks in Hydrology [2000b]:

In contrast, ANNs can be trained on input-output data pairs with the hope that they are able to mimic the underlying hydrologic process.

Figure 9   The basic principle of the well-known multilayer perceptron

So, the majority of the studies concerning MLPs paint a glorious picture as to their merits and efficiency. The following critique underlines their inherent drawbacks which the new GAIN-P methodology will overcome:

- All the summary articles finish with the résumé that there are, as yet, still no official procedure guidelines for ANN-based modelling in hydrology. The MLP applications are generally case studies where the application is focused on particular areas, occurrences and/or processes. As far as the drafting of a training program for the MLP is concerned, the given guidelines are generally very vague and not at all transparent. In addition, the conclusions of the various scientific articles are difficult to compare and/or match with each other and, what is more, it is not possible to transfer the chosen set of network parameters or training methods for helping to solve your own individual problem. Last but not least, due to the subjectivity contained in the architectural design of the MLP, it is practically impossible to repeat the investigation accurately step by step. For these reasons, Dawson and Wilby [2001] proposed a framework for the modelling of ANN-based rainfall-runoff models.
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• MLPs are mostly used as rainfall-runoff models. It is precisely in this area that the MLPs' shortcomings, and in particular their utmost dependence upon the quality and relevance of the training data, become apparent. This is best illustrated by the example of predicting rare extreme hydrologic events; as the training records generally do not contain such data and as the extrapolation capacity of MLPs is dubious, the forecasting of such events cannot succeed [Minns and Hall, 1996].

• In the few studies undertaken, radial basis function networks (RBFs) compared favourably with MLPs as regards the prediction performance [Mason et al., 1996; Jayawardena and Fernando, 1998; Wilby, 1999]. Comparisons between MLPs and other network architectures, as for e.g. self-organizing maps [Kothari and Islam, 1999] or recurrent networks [Zhu et al., 1998], do not exist because the latter have only been applied to very specific tasks. This gives rise to Dawson and Wilby's [2001] declaration that there is also an urgent need for more inter-model comparisons and rigorous assessment of ANN solutions ....

Although MLPs have been used for a wide variety of applications in hydrology, their application with respect to solving the inverse problem (see section 2.1) has not yet been realized but will be performed subsequently in this case study.

3.1.2 The problem-adapted self-organizing map architecture: the SOM-MIO

By means of non-supervised learning, on the basis of a self-organizing process, Kohonen [1982] proposed a neural model which generates so-called Self-Organizing Maps (SOM) as an alternative approach to artificial intelligence techniques. This basically different network architecture paves the way for overcoming the aforementioned essential drawbacks associated with MLPs.

Self-organizing maps in hydrological applications

It comes as a surprise to learn that up until now self-organizing maps have only played a subordinate role in publications which deal with the application of neural networks in hydrology. This unfortunate state of affairs does, however, facilitate the task of listing and discussing all the relevant publications which have already appeared in this vein. In the majority of cases, the standard SOM is used as a tool for the classification or clustering of pre-process data. Cai et al. [1994], for example, use a standard SOM for classifying flow conditions in the unsaturated soil-water zone. Their experiments resulted in a good correspondence between the SOMs' classifications and visual observation. Other fields of application for the standard SOMs include the reduction of the dimensionality of data resulting from remote sensing [Kothari and Islam, 1999] or, again, for the pre-classification of training data for multilayer perceptron networks [Bowden et al., 2002]. Rizzo and Dougherty [1994] can be considered to have taken the first step towards enabling standard SOMs to produce discrete output values. They use the Counter Propagation Network (CPN) architecture as proposed by Hecht-Nielsen [1987]; this combines a SOM with a so-called 'Grossberg layer' for the inverse calculation of the saturated hydraulic conductivity based on the observation of groundwater flow. Sadly, the implementation of the CPN architecture exhibits a relatively restricted neighbourhood cooperation within the SOM layer and, thus, the Grossberg layer is only able to generate discrete output values. Using data from remote sensing, Hsu et al. [1997] then developed a modified CPN architecture for rainfall forecasting. For calculating the output, their model employs a weighted sum of the best matching unit (BMU) and
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neighbouring neurons and, in so doing, places more importance upon neighbourhood cooperation.

Hydrological research is currently at pains to equip SOMs with extensions for a continuous output. Along these lines, Ranayake et al. [2002] combine SOMs and MLPs for evaluating the saturated hydraulic conductivity and the dispersion coefficients for matter and solution transport in the ground water. With this hybrid architecture, the SOM is used for a rough estimate of the range of parameter values and, depending on the subsection concerned, activates a specially trained MLP, which then conducts the final estimate of the parameter values to be used. This relatively cumbersome hybrid architecture even managed to work reliably when, on occasion, distorted values were supplied for the input. In 2002, Hsu et al. likewise introduced the Self-Organizing Linear Output Mapping Network architecture (SOLO), which was tested in an application with rainfall-runoff modelling. The SOLO architecture consists of two SOM layers of similar dimensions. The input layer is a standard SOM, whereby linear regression models are assigned to the neurons of the output layer instead of the standard characteristic vectors. The training program combines non-supervised learning with the calculation of the regression parameters, by minimizing the least squared error between the network output and the training data. As regards the application for the rainfall-runoff model, and after comparing the results of the two different approaches, the SOLO architecture proved itself superior to an MLP. In addition, the SOLO architecture can provide its user with information regarding the current status of the rainfall-runoff process (something which the MLP is not able to do) and, furthermore, it is more robust during training. One of the techniques related to the SOLO architecture is the aforementioned local linear maps, whereas section 4.2 gives a more detailed description of Schütze and Schmitz's [2003] application of a SOM for irrigation control.

The most up-to-date type of architecture, to be found in the latest publications by Hsu et al. [2002] and Schütze and Schmitz [2003], not only perform extremely well, they also show great potential for applications in hydrology, including for the control of irrigation processes. These methods help to solve the mystery behind these so-called black box neural networks. An appropriate evaluation of the parameters (feature vectors) at the individual computational knots enables a comprehensive insight into the SOM's learning behaviour during training and, similarly, into how it arrives at its calculations when being used either as a hydrological model or tool for controlling purposes.

3.1.2.1 Mathematical principles of training self-organizing maps

The structure and training algorithm of Kohonen's original SOM are of fundamental significance and are therefore highlighted first of all. Contrary to the network architectures already introduced, SOMs only have one input and one hidden layer (Figure 10(a)). The input layer transfers the input signals to the different neurons of the hidden layer. Contrary to MLP and RBF, the classical SOM does not guarantee an output vector \(\mathbf{y}'\) as a response to an input signal \(\mathbf{x}\). Instead, it responds to the activation by activating one particular neuron of the hidden layer, the so-called 'winner', i.e. the BMU. For a one dimensional SOM, Figure 10(b) displays the neuron with index \(c\) as the BMU for an arbitrary input signal \(\mathbf{x}\).

The SOM network used in this investigation consists of \(l\) neurons organized on a regular two-dimensional grid. A \((n + m)\) dimensional weight vector \(\mathbf{m} = (m^1,...,m^{n+m})\) is assigned to each neuron, where \(n = \text{dim}(\mathbf{x})\) and \(m = \text{dim}(\mathbf{y})\) denote the dimensions of the sample input and sample output, respectively. Thus, contrary to MLP and RBF networks, the
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input signal of the SOM always consists of both input and output vectors \((\tilde{x}, \tilde{y})\) which are specified in detail in section 2.1. The neurons are connected to adjacent neurons by a neighbourhood relationship which defines the topology or the structure of the SOM. We use a hexagonal grid for the neighbourhood relationship \(N_i\) and a two-dimensional structure of the self-organizing map. Generally, the SOM is trained iteratively. Each iteration \(k\) involves an unsupervised training step using a new sample vector \(\tilde{x}_{SOM}\) and the weight vectors \(\vec{m}_i\) were modified according to the following training procedure:

**Begin: Training the SOM**

**Initialization of the SOM:** Choose random values for the initial weight vectors \(\vec{m}_i\).

**Begin: Iteration \(k\)**

**Iteration Step 1: Search for the Best Matching Unit (Winner)**

At each iteration \(k\) one single sample vector \(\tilde{x}_{SOM}(k)\) is randomly chosen from the input data set and its distance \(\varepsilon_i\) to the weight vectors of the SOM is calculated by:

\[
\varepsilon_i = \|\tilde{x}_{SOM}(k) - \vec{m}_i\| = \sum_{j=1}^{N_{SOM}} (x_{SOM}^j(k) - m_i^j)^2
\]

The neuron whose weight vector \(\vec{m}_i\) is closest to the input vector \(\tilde{x}_{SOM}(k)\) is the 'winner', i.e. the BMU \(c\), represented by the weight vector \(\vec{m}_c(k)\):

\[
\|\tilde{x}_{SOM}(k) - \vec{m}_c(k)\| = \min\{\|\tilde{x}_{SOM}(k) - \vec{m}_i\|, i = 1, 2, ..., l\}
\]

**End Iteration Step 1**

**Iteration Step 2: Weight vectors update**

After finding the BMU \(c\), the weight vectors of the SOM are updated. Thus, the BMU \(c\) moves closer to the input vector in the sample space. Figure 11 shows how the reference
vector $\hat{m}_i(k)$ of the BMU and its neighbours move towards the sample vector $\hat{x}_{\text{SOM}}(k)$. Figures 11 (a) and (b) correspond to the situation before and after updating, respectively. The rule for updating the weight vector of unit $i$ is given by:

$$\hat{m}_i(k + 1) = \hat{m}_i(k) + \alpha_i(k) h_i(k) [\hat{x}_{\text{SOM}}(k) - \hat{m}_i(k)]$$

(18)

where $k$ denotes the iteration step of a training procedure, $\alpha_i(k)$ is the learning rate at step $k$ and $h_i(k)$ is the so-called neighbourhood function which is valid for the actual BMU $c$. $h_i(k)$ is a non-increasing function of $k$ and of the distance $d_{ci}$ of unit $i$ from the BMU $c$. The Gaussian function is widely used to describe this relationship:

$$h_i(k) = e^{-d_{ci}^2/2\sigma^2(k)}$$

(19)

$\sigma$ is the neighbourhood radius at iteration $k$ and $d_{ci} = \|\vec{r}_c - \vec{r}_i\|$ is the distance between map units $c$ and $i$ on the map grid. The neighbourhood radius $\sigma$ corresponds to the neighbourhood relationship $N_i$.

End Iteration Step 2

End: Iteration $k$

End: Iteration $n_D$

End: Training of the SOM

Steps 1 and 2 are repeated for one or many times the amount $n_D$ of sample vectors until convergence is achieved. Convergence implies that $h_i \rightarrow 0$ for $k \rightarrow \infty$ and thus, depends on the function of the neighbourhood radius $\sigma(k)$. A common choice is an exponential decay described by Ritter et al. [1992]:

![Figure 11](image-url)  

(a) SOM before the update  

(b) SOM after the update

Update the BMU and its neighbours towards the sample vector $\hat{x}_{\text{SOM}}$ marked $\text{x}$
\[ \sigma(k) = \sigma(0)e^{-k/k_{\text{max}}} \]  \hspace{1cm} (20)

The learning rate \( \alpha_s(k) \) should also vary with the increasing number of training steps as indicated in Equation 18. Kohonen [2001] suggested starting at an initial value \( \alpha_s(0) \) with a value close to 1 and then decreasing gradually with an increasing number of training steps \( k \), e.g.

\[ \alpha_s(k) = \alpha_s(0)e^{-k/k_{\text{max}}} \]  \hspace{1cm} (21)

The cooperation between neighbouring neurons, a unique feature of the SOM algorithm, ensures a fast convergence and a high accuracy in approximating functional relationships. Even though the exponential decays described in Equations 20 and 21 for the neighbourhood radius \( \sigma(k) \) and the learning rate \( \alpha_s(k) \) are purely heuristic solutions, they are adequate for a robust formation of the self-organizing map [Kohonen, 2001].

3.1.2.2 The new SOM-MIO

Originally, the SOM is used as a tool for solving classification problems, e.g. as feature extraction or recognition of images and acoustic patterns. When using the SOM for these tasks, the most important step before application is the interpretation of its final internal structure, i.e. the labelling of the network units with a certain class. The result of an operation of a classic SOM then represents discrete information, e.g. a certain phoneme in speech recognition or character in optical character recognition (OCR) represented by the BMU.

In order to broaden the possible range for application in water resources, we now expand the SOM principle. When applying a trained SOM, we introduce a new interpolation method which generates multiple continuous output information. This leads to the new SOM-MIO architecture which arranges the data vectors during application into two predefined parts, depending on the underlying problem. Rearranging the original data vectors allows switching between the different mapping functions provided by the SOM-MIO. For example, consider a sample vector \( \vec{x}_{\text{SOM}} \) with three components \( (x^1, x^2, x^3) \). This results in three options for operating the SOM-MIO:

1) \( \vec{x} = (x^1, x^2), \vec{y} = x^3 \)
2) \( \vec{x} = (x^2, x^3), \vec{y} = x^1 \) and
3) \( \vec{x} = (x^1, x^3), \vec{y} = x^2 \).

\( \vec{y} \) denotes the required output which is not available during application.

Interpolation using the centre of gravity of the \( n \) best matching units (IBMU)

The new architecture of the SOM-MIO allows for a calculation of the required output value \( \vec{y} \) using an adapted interpolation method. During the application, step 1 of an iteration in the original SOM learning algorithm is performed, but with a small modification. In contrast to the training procedure, only the predefined \( \vec{x} \) part of \( \vec{x}_{\text{SOM}} \).
\( \tilde{x}_{SOM} D_{x} \), with \( D_{x} = \text{diag} \{ d_{i} \}, d_{i} = \begin{cases} 1 & i = 1 \ldots n \\ 0 & i = n + 1 \ldots n + m \end{cases} \) (22)

is compared with the corresponding part of the weight vector \( \tilde{m}_{i} \) of each neuron:

\[ \left\{ \| \tilde{x}_{SOM} - \tilde{m}_{i} \| \right\}_{i=0}^{n_{\text{max}}} = \min_{i=0}^{n_{\text{max}}} \| \tilde{x}_{SOM} D_{x} - \tilde{m}_{i} D_{x} \|, i = 1, 2 \ldots l. \] (23)

After finding a given number of best matching units \( n_{\text{bmu}} \) (see Figure 12) by using Equation 23, their respective quantization errors \( \varepsilon_{i} \) are calculated (cf. Equation 16). The reciprocal of the relative quantization errors are treated as weights in a linear subspace of \( \mathbb{R}^{n} \). The quantization errors are determined by the components \( \tilde{m}_{i} D_{x} \) of the \( n_{\text{bmu}} \) units. The idea is now to use the 'centre of gravity' of the \( n_{\text{bmu}} \) output components \( \tilde{m}_{i} D_{y} \) for determining the interpolated output value \( \tilde{y} \). The response \( \tilde{y} \) of the SOM-MIO is evaluated by:

\[ \tilde{y} = \sum_{i=0}^{n_{\text{max}}} \frac{1}{\varepsilon_{i}} \cdot \tilde{m}_{i} D_{y} \] (24)

with

\[ D_{y} = \text{diag} \{ d_{i} \}, d_{i} = \begin{cases} 0 & i = 1 \ldots n \\ 1 & i = n + 1 \ldots n + m \end{cases}. \] (25)

---

**Figure 12** Determination of the BMUs responsible for generating output information for a sample vector \( \tilde{x}_{SOM} D_{x} \) marked \( \times \)

\( D_{x} \) and \( D_{y} \) must be a priori defined according to the chosen mapping function in order to select input or output components of reference and data vectors. For example, operating the SOM-MIO with option (1) above requires
By specifying the appropriate pattern of the matrix diagonals in $D_x$ and $D_y$, the same SOM-MIO can be used for different mapping tasks, i.e. either for approximating a numerical model or for solving the corresponding inverse problem. This is a unique feature of the SOM-MIO amongst all other ANN architectures. Furthermore, the used interpolation scheme is a robust calculation method because the output values always lie within the range which is given by the training data.

3.2 Evolutionary algorithms (EA)

In 1848 Charles Darwin published his masterpiece 'On the origin of species by natural selection'. With his theory, Darwin was able to show that evolution works on the basis of variation and selection. Variation is the term for the phenomenon which produces differences between parents and offspring, whereas selection determines which type of individuals survive and procreate. The observation that those species who best adapt to their natural surroundings are the ones which flourish led, during the 1960s, to attempts at developing approaches for efficient optimization processes. Attention was focused on the optimization of objective functions which were not able to be optimized on the basis of traditional optimization procedures. Due to their significance in the development of successfully applicable evolutionary algorithms (EA), the following methodologies are relevant:

- the evolutionary programming
- the evolutionary strategies
- genetic algorithms.

Despite the fact that the above-mentioned algorithms have many things in common, for a long time they were developed and dealt with separately. It was not until the middle of the 1980s that first attempts were made to reduce the methodologies to a common uniform theoretical basis. This led in the 1990s to the development of a multitude of innovative evolutionary algorithms.

Algorithm 1 General setup of evolutionary algorithms.

1. $k = 0$
2. initialize $X^0 = \{x_1^0, ..., x_n^0\} \in I^p$
3. evaluate $X^0 : \{\Phi(F(x_1^0)), ..., \Phi(F(x_n^0))\}$
4. while (convergence($X^k$) $\neq$ true) do
   1. recombine : $X''^k = rec(X^k)$
   2. mutate : $X''^k = mut(X''^k)$
   3. evaluate : $X''^k : \{\Phi(F(x_{1}''^k)), ..., \Phi(F(x_n''^k))\}$
   4. select : $X^{k+1} = sel(X''^k \cup A)$
   5. $k = k + 1$
5. end
The population \( X \) comprises in each generation \( k \) \( \mu \) individuals \( x_i^k \), \( i = 1 \ldots \mu \). For each generation step \( \lambda \) new individuals are formed with \( \lambda \geq \mu \). The methodology involves operators which work in accordance with the law of natural evolution. Some of the operators are:

- Recombination \( \text{rec} \). By combining together several individuals from the preceding generation, new individuals are generated.
- Mutation \( \text{mut} \). By altering the characteristics of a single individual, other new individuals are formed. Depending on the type of procedure being used, this can also apply for individuals which were originally created by the recombination operator.

On the basis of the selection \( \text{sel} \) individuals \( \mu \) are chosen from amongst the set of the newly created individuals. It may happen that certain individuals are selected more than once so that it can even be the case that selection takes place exclusively amongst those \( \mu \) of the newly created individuals. With some types of procedures, the set which contains the individuals for selection can also include, besides the \( \lambda \) newly created individuals, other individuals from past generations. These are grouped together in a set.

The particular specifications of the individual operators, which follow the principles to be found in genetics, and the data structure of the different individuals are not stipulated. These characteristics can be arbitrarily chosen from within a large framework, on condition that the recombination and mutation strategies are adapted to suit the data structure concerned. Data structures which are often used include:

- the representation of the individuals as binary strings, which contain the variables yet to be optimized in coded form
- the representation of the individuals as vectors of real numbers, which generally mirror in an unchanged form the specifications of the variables yet to be optimized.

This representation of the individuals as a binary string is particularly relevant for application in genetic algorithms and derives its strategies from DNA principles. As far as the processing of continuous optimization problems is concerned, due to the necessary discretization, this binary string method is associated with many drawbacks. Amongst others, the problems include:

- the choice of appropriate discretization steps
- the extreme length of the strings which is an unavoidable feature of high dimensional optimization problems and/or when a very fine discretization is necessary
- difficulties when it comes to adequately considering the different weights of the individual bit positions within the string
- the feasibly large Manhattan distance between two neighbouring real numbers. This unfortunately leads to a substantial discrepancy between the problems to be solved and the corresponding code.

When coding vectors of real numbers, these types of problems either do not appear at all or they are not significant. Many empirical investigations of practical problems indicate a substantially improved suitability of this type of solution strategy for continuous optimization problems [Nissen, 1997; Pohlheim, 1998; Bäck, 1996]. Because the difficulties encountered in optimizing irrigation schedules is a continuous optimization problem, the subsequent development is based upon a real mapping of the problem variables.
3.2.1 Data structure

A single irrigation schedule $s$ is apportioned to each individual $x^k_j$ of the population $X^k$ of the $k^{th}$ generation. Each irrigation schedule consists of a group of two tuples $(t_i, \hat{I}_i)$, where each single tuple contains the data of an individual irrigation event. These data feature:

- the time of the irrigation $t_i$
- the volume of water to be applied $\hat{I}_i$.

The number of the tuples $x^k_j$ to be found in the group is determined by the number $n^k_j$ of the single irrigation events $(t_i, \hat{I}_i); i = 1...n_j$ prescribed in the schedule. Thus, the individuals of the population $X^k$ of a generation $k$ can be given as:

$$X^k = \{x^k_j\}_{j=1,...,\mu} = \{(t_i, \hat{I}_i)_{i=1,...,n^k_j}\}_{j=1,...,\mu}$$ \hspace{1cm} (27)

The entire group $X$, i.e. all those individuals which come into being during the optimization, can be formed by bringing together all the individuals of each and every generation.

$$X = \{X^k\}_{k=1,...,\mu} = \{x^k_j\}_{j=1,...,\mu} = \{(t_i, \hat{I}_i)_{i=1,...,n^k_j}\}_{j=1,...,\mu}$$ \hspace{1cm} (28)

3.2.2 Basic structure of the algorithm

The basic structure of the algorithm is shown in Figure 13 and, in certain aspects, deviates from the standard set-up of evolutionary optimization algorithms. These deviations include:

- a change in the order in which the individual operators are dealt with. During each single generation step the selection becomes the first operation to be carried out instead of coming at the end. By doing this, it becomes possible to neglect additional function evaluations within the initialization; this results in a decrease in the overall number of the necessary function evaluations.
- an additional reconstruction step for exploiting already available knowledge in order to be able to guarantee compliance with the complex constraints.

The following sections describe the way in which the algorithm's individual components operate.

3.2.3 Initialization

During the initialization, each individual in the initial population must be assigned with sensible values. This implies that the generated individuals:

- should be distributed as uniformly as possible in the search space in order to avoid an unintentional premature convergence, which could occur due to the high concentration of the initial population in the small sub-space.
- should not violate any of the constraints.
Methods: the modules of GAIN-P

The variable number of irrigations $n$ poses a particular problem for the initialization. It gets more complicated to assign the individuals with arbitrarily located vectors of a constant length, which is what normally happens when initializing evolutionary algorithms. Instead of this, for each individual $x_i^0 : i = 1 \ldots \mu$ of the initial population, the number of the irrigations between 1 and the maximum possible number of irrigations $n_{\text{max}}$ is uniformly distributed on a probabilistic basis. In a final step, an irrigation time is determined, likewise uniformly distributed, for each irrigation throughout the growing period, and the volume of water $I_{\text{min}}$ and $I_{\text{max}}$ to be applied is also prescribed.

Algorithm 2 Initialization.

\begin{verbatim}
for ($j = 1; j \leq \mu; j++$)
  $x_j^0 = \{\}$
  $n_j^0 = U(1, n_{\text{max}})$
  for ($l = 1; l \leq n_j^0; l++$)
    $x_j^0 = \{x_j^0, U(0, t_{\text{max}}), U(I_{\text{min}}, I_{\text{max}})\}$
  endfor
endfor
\end{verbatim}

It may be the case that the thus generated schedules end up violating the constraints. For this reason, a reconstruction of the population is carried out at the end of the initialization, with the help of the reconstruction operator. As a consequence of the reconstruction, all the invalid individuals are transferred into valid ones.
3.2.4 Selection

Due to the fact that the selection takes place in the search space only on the basis of the value of the objective function, i.e. without taking the inner structure or the position of the individuals into account, it is quite a straightforward matter to adopt the already proposed selection strategies. For this purpose we rely on the Tournament selection because it:

- is extremely robust
- has already often been used successfully in practice
- is straightforward to implement and to control, and
- is in a position to use pareto-optimal selection for optimizing multi-dimensional objective functions. In terms of the procedure's potential for future development, this is not without significance.

Thus, for each individual $x_j^{(0)}$ of the new generation, the best-suited individual $m$ is chosen, on a probabilistic basis, from amongst the individuals of the preceding generation $k - 1$. If the new individual is to be generated via the recombination, which is the case with the recombination probability $p_r$, a second parent $x_j^{(0)}$ must be determined.

**Algorithm 3 Selection.**

```
for (j = 1; j ≤ μ; j + +)
    $x_1^{(1)} = \text{argmax}(Y((x_{N(1,μ)}), \ldots, (x_{N(1,μ)})))$
    if (U(0, 1) > $p_r$)
        $x_1^{(2)} = \text{argmax}(Y((x_{N(1,μ)}), \ldots, (x_{N(1,μ)})))$
    endif
endfor
```

3.2.5 Recombination

By combining together two individuals, the recombination produces a third, new individual. As far as the entire population is concerned, the percentage of the individuals produced by recombination is determined via the recombination probability $p_r$. Due to the fact that the number of irrigated events can differ between the two parent individuals, it is not possible to directly adopt one of the recombination operators. Instead, the recombination operator must be altered to suit the structure of the data.

Because plant water uptake is time-dependent, with respect to recombination it makes sense to stick with the relationship between the irrigation time and volume which was learnt by the two parents. This can be achieved by creating the offspring individual out of a selection of irrigation tuples, which themselves are chosen from the combined total of the parents own irrigation tuples. On the basis of the probability $p_w$, a concrete tuple is selected and adopted in its original, unchanged form.
Once the recombination is completed, the new individual $x_j$ possesses an arbitrary combination of the parents' individual irrigation events, whereby the relationship between the different irrigation times and volumes remains as it was. In order to avoid a tendency towards irrigation schedules which prescribe fewer and fewer irrigation events, the value for $p_w$ should be chosen within the range of 0.5 to 1. With the choice $p_w = 1$ the operator loses its stochastic character. When this is the case, the new individual possesses all the irrigation events of both parents combined.

**Algorithm 4** Recombination.

\[
\begin{align*}
x_j'' &= \{\} \\
\text{for } (i = 1; i \leq n_j^{(1)'}; i ++) \\
\quad &\text{if } (U(0, 1) \leq p_w) \\
\quad \quad x_j'' &= \left\{ x_i'', \left\{ t_i^{(1)'}, \hat{I}_i^{(1)'} \right\} \right\} \\
\text{endif} \\
\text{endfor} \\
\text{for } (i = 1; i \leq n_j^{(2)'}; i ++) \\
\quad &\text{if } (U(0, 1) \leq p_w) \\
\quad \quad x_j'' &= \left\{ x_i'', \left\{ t_i^{(2)'}, \hat{I}_i^{(2)'} \right\} \right\} \\
\text{endif} \\
\text{endfor}
\]

### 3.2.6 Mutation

For each case, the mutation works with one individual at a time only. Because this makes it possible to consider an individual's personal, distinct variables independently of each other, the Gauss mutation (which was introduced earlier) is able to be applied more or less in its original, unchanged form.

For all the irrigation times $t_i$ and irrigation volumes $\hat{I}_i$ of an irrigation schedule, the mutation takes place by adding a normally distributed random parameter, which has to be freshly determined for each variable to be mutated.

Different crops react differently to changes made in the irrigation timing and/or in the irrigation water volumes. For this reason, with regard to the controlling of the mutation, a distinction is made between the variances for the mutation of the irrigation times $\sigma_{t_i}$ and between the variances for the mutation of the irrigation volumes $\sigma_{\hat{I}_i}$. These parameters are subsequently denoted as mutation parameters.
Irrigation control: towards a new solution of an old problem

3.2.7 Reconstruction

Unfortunately, either as a consequence of mutation or due to recombination, irrigation schedules can materialize which end up violating the constraints mentioned in section 2.3. These constraints include:

- a compliance with the minimum time intervals between the different irrigation events $\Delta t_{\text{min}}$
- an adherence to the limits regarding the total irrigation water volume $I_{\text{max}}$
- keeping above the limits with respect to the minimum amount of irrigation water to be used per irrigation event $\hat{I}_{\text{min}}$.

In order to guarantee compliance with these constraints, the reconstruction operator transfers invalid individuals, i.e. those who violate the constraints, into valid individuals. The reconstruction of the invalid individuals is based on the methodology of the backwards coupled repair function. This means that the invalid individual is substituted in the population by the valid individual which was created by the reconstruction.

Reconstruction involves two steps:

First reconstruction step

A compliance with the minimum time intervals between the individual irrigation events $\Delta t_{\text{min}}$ is guaranteed because if separate irrigation events are planned too soon one after another, these are combined together to form one irrigation event only. This happens on the basis of a kind of micro-recombination, which considers two individual irrigation events which are timed too closely together in the schedule. These two irrigation events are removed from the schedule and substituted by a new one. The resulting new irrigation event retains the scheduled timing of the earlier of the two formerly planned irrigations, but its revised irrigation water volume is calculated by combining together the volumes of the two originally scheduled events.

This first step is very important in order to ensure keeping to the minimum time intervals between the individual irrigation events for the entire duration of the growing period. Starting with the first irrigation event, the aforementioned procedure must be constantly repeated for the duration of the schedule until the periods of time between the irrigation events are all sufficiently long enough not to fall short of the prescribed minimum limits.

Working on the assumption that soil has practically perfect water storage filling and emptying characteristics, at least for short periods of time, this methodology ensures the best compromise in terms of water availability for the plants. A delay in the timing of the first irrigation event

\[ x_j'' = \{ \}
\]

\[ \text{for } (i = 1; i \leq n_j^{(k)}, i++ ) \]

\[ t_i'' = t_i'' + N(0, \sigma_i) \]
\[ I_i'' = I_i'' + N(0, \sigma_j) \]
\[ x_j'' = \{ x_j''; (t_i'' , i'') \} \]

endfor
does not expose the plants to unnecessary water stress, and neither does the water shortage caused by the cancellation of the second originally scheduled irrigation.

**Algorithm 6** First reconstruction step.

```
for (i = 1; i ≤ (n'' - 1); i + +)
    if (abs (t_i'' - t_{i+1}'') < Δt_{min})
        t_i'' = min(t_i'', t_{i+1}'')
        I_i'' = I_i'' + I_i''
        remove a_i''+1
        n_j'' = n_j' - 1
        end = 1
    endif
endfor
```

**Second reconstruction step**

An assurance is likewise given with respect to keeping within the limits of the total irrigation water volume for the season I_{max} and the minimum amount of water to be used per irrigation event I_{min}. This is brought about by averaging the sum of the individual irrigation water volumes with the sum total of the irrigation water volume available. The smallest irrigation event, which, in water volume terms, is even smaller than the authorized minimum amount I_{min} is omitted from the schedule. This procedure is repeated until each and every individual irrigation event consumes at least the minimum amount of the prescribed irrigation water volume I_{min}.

Due to the standardizing of the irrigation schedule, the schedule can exploit the exact amount available of the total irrigation water volume and, thus, the optimization algorithm no longer needs to calculate the maximum amount of water possible. This methodology is suitable when assuming a deficit irrigation strategy, as it is to be expected that under optimal scheduling conditions, if more water is made available to the plants, this should lead to an improved crop yield. A maximum crop yield can, thus, only be achieved if every drop of the available irrigation water is used.

**Algorithm 7** Second reconstruction step.

```
I_i'' = I_i'' * \frac{I_{max}}{\sum_{i=1}^{n_j''} l_i''}
for i = 1..n_j''
while min(I_i'') < I_{min}
    remove arg min(I_i'')
    I_i'' = I_i'' * \frac{I_{max}}{\sum_{i=1}^{n_j''} l_i''}
    for i = 1..n_j''
endwhile
```
As far as these two repair steps are concerned, the first and/or the second can help to reduce the number of irrigation events. In terms of the number of irrigations \( n \), the reconstruction operator is therefore not just to be seen as a simple repair function. The operator interacts with the recombination and adopts the role of search operator for optimizing the number of the individual irrigation events.

### 3.2.8 Extension of the selection with the help of a steady state procedure

The efficiency of the evolutionary algorithm depends heavily on the explorative components of the mutation operator. However, in this context, the mutation rate must not be allowed to exceed certain limits, so as not to jeopardize the convergence of the procedure. In spite of this risk factor, the technique called the 'steady state procedure' makes it nevertheless possible to apply higher mutation rates. At the same time, by employing the steady state procedure, this guarantees being able to retain the best individual. Due to the fact that the best individual of a particular generation is certain to win the tournament for which it is intended, it is sure to feature again in the next generation. As for the fate of the other individuals, relatively speaking, the larger the size of the tournament and the lower the rank of the individuals concerned, the less likely it becomes that the remaining individuals will be taken up by the subsequent generation in their original, unchanged form. By deviating in this manner from the normal procedure, it is possible to avoid having to use an additional control variable, which is responsible for determining the number of the individuals to be taken over unchanged into the next generation. This can be done without jeopardizing the positive features of either the tournament or the steady-state-procedure.

### 3.3 Physically based modelling of the surface-soil-crop system

This section begins with a more general part introducing the principles of irrigation modelling. The following sections contain a detailed description of the model system FIM for the physically based simulation of furrow irrigation throughout an entire growing season. Firstly, the FIM modules of the surface flow, the infiltration and subsurface flow, and the crop growth are depicted separately. Next, the surface module and the subsurface module are iteratively coupled and a highly effective numerical solution of the resulting system of nonlinear equations is presented. In another section, the coupling of the crop growth module and the subsurface module is described. Finally, the various modules are integrated to the model system FIM by a comprehensive time management and event control unit. The FIM model parameterization is supported by a user-friendly graphical user interface (GUI) which is portrayed at the end of this section.

#### 3.3.1 Principles of irrigation modelling

The modelling of irrigation systems contributes significantly towards improving both the system's efficiency and performance. A precondition for these improvements is the realistic mapping of the main physical water transport processes within the system throughout the entire growing season. The more challenging task of simulating the water flow in furrow irrigation systems is dealt with in this contribution.
Methods: the modules of GAIN-P

Any furrow irrigation model should employ an overall water balance which is a useful concept for characterizing, evaluating or monitoring surface irrigation systems [Walker and Skogerboe, 1987]. Three interacting regions are distinguished within the furrow irrigation system, namely the surface, the soil or subsurface and the crop. The principles of modelling the water flow within these regions are subsequently outlined and a number of performance criteria for evaluating irrigation systems are described at the end of this section.

3.3.1.1 Water balance

The water balance is a fundamental base for modelling the water transport processes and, whichever degree of model sophistication is chosen, must comply at all times with the system's boundaries. Irrigation systems have generally well-defined boundaries. The topographical field borders are the vertical boundaries of the system. In the horizontal plane, the system is bounded by the soil surface at the top and a certain soil depth at the bottom which may be, for example, the rooting depth of the plants or the groundwater table. Figure 14 shows a furrow irrigation scheme, with the boundary nodes A, B, C, D, E, F, G, H (nodes F and G are located below nodes B and C but are not visible in the plot). The horizontal boundaries of the flow domain are defined by the two planes ABCD and EFGH. The vertical boundaries are fixed by the planes AEBF, BFCG, CGDH and AEDH. Due to the symmetry of adjacent (idealized) furrows, the vertical boundaries (planes AEBF and CGDH) are often drawn along the ridges of both sides of a single furrow, as shown in section 1.3.1.

The principle of continuity requires that inflow minus outflow equals the change in storage, $\Delta S$, within the defined boundaries of a system [Walker and Skogerboe, 1987]. In the case of the

![Figure 14 Water balance parameters for a furrow-irrigated field](image-url)
furrow irrigation system (Figure 14), the continuity equation can be written as

\[ [V_{in} + V_P + V_{CR}] - [V_{EA} + V_{TA} + V_{DP} + V_{surf} + V_{out}] = \Delta S \]  

(29)

in which

\[ V_{in} = V_{surf} + V_{inf} + V_{out} \]  

(30)

with

- \( V_{in} \) = the irrigation water inflow volume
- \( V_P \) = the precipitation volume
- \( V_{CR} \) = the volume of capillary rise
- \( V_{EA} \) = the evaporation volume
- \( V_{TA} \) = the transpiration volume
- \( V_{DP} \) = the volume of deep percolation
- \( V_{surf} \) = the volume of the surface water body
- \( V_{out} \) = the surface runoff volume
- \( V_{inf} \) = the volume of infiltration.

Equation 29 comprises the following assumptions:

- Water from rainfall events, \( V_P \), completely enters the soil by infiltration, i.e. rainfall is not intercepted by the plants and does not partially leave the field by surface runoff.
- There is no lateral water flow from/to the subsurface flow domain.

The groundwater contribution to the root zone soil moisture \( V_{CR} \) can be ignored if the groundwater table is more than 7 m below the ground surface for heavy (clay) soils and more than 3 m for light (sandy) soils, unless the crop roots nearly extend down to the groundwater table level [Walker and Skogorge, 1987].

However, the accurate prediction of the several components of the volume balance is extremely difficult and thus makes irrigation modelling a very complex task.

3.3.1.2 Surface flow

In surface-irrigation modelling, much emphasis has been placed on the description of surface water flow. The advance of the surface water body determines the wetting or infiltration opportunity time at each location along the furrow length. Infiltration opportunity time and flow depth are important parameters controlling local infiltration for a given soil.

Surface flow characteristics change considerably in time and space during the different phases of the irrigation. Bearing this in mind, this section distinguishes four different phases of an irrigation event. The mathematical modelling of surface flow utilizes hydrodynamic flow equations which are subsequently described.

**Flow characteristics**

During a furrow irrigation event, water moves over the land surface in open channel flow. In contrast to borders and basins, the geometry of irrigation furrows, i.e. the channel walls, have significant influence on flow-retarding and infiltration. A part of the surface water leaves the surface as infiltration in the soil. The rate of infiltration at any point of the furrow typically decreases with increasing wetting time. As a consequence, the flow rate and depth at any point in the furrow will gradually increase with time, as long as the discharge at the inlet
remains constant. Additionally, the flow rate decreases with the distance from the furrow inlet. Once the advancing front has reached the lower end of the furrow, the water either leaves the field as surface runoff or it develops surface storage if the furrow end is blocked to prevent runoff. The inflow discharge is stopped at some later point in time when sufficient water has been supplied to the soil to meet crop water requirements and/or other irrigation targets (e.g. leaching requirements). Surface flow continues after ‘cut-off’, but with decreasing water depth and velocity. Once the flow depth at the field inlet has reached zero, a drying or recession front moves down the channel until it reaches the lower field end. Irrigation is now complete.

In contrast to the irrigation advance phenomenon, it is very difficult to observe the location of the ‘receding wave’ due to variations in the intake rate, the field slope and the roughness. Recession often occurs simultaneously over a fairly wide reach of the field (say 1 to 10 m) and two or more observers will locate recession at the same time, but at substantially different locations [Walker and Skogerboe, 1987].

The surface flow is characterized by different flow regimes at different times and places in a single irrigation. For instance, the flow near the field inlet at the beginning of the irrigation is markedly different from the shallow, low-velocity flow that occurs during recession [Bassett and Fangmeier, 1980]. Another example is the flow at the advancing wave tip which is highly turbulent. At the tip, the water level is directed towards the vertical. However, many flow models neglect these differences and are based solely on the one single flow regime which dominates the major part of a typical irrigation.

Because of water infiltration into the soil, the channel flow can be characterized as being gradually varied and unsteady free surface flow for a major part of the irrigation [Walker and Skogerboe, 1987; Wallender and Rayej, 1990]. In irrigated furrows, the observed flow velocity is relatively low [Clemmens, 1979] and the flow is sub-critical, since Froude numbers are usually well below unity [Bassett and Fangmeier, 1980].

Phases of an irrigation event
Four different hydraulic phases of an irrigation event are distinguished according to the temporally and spatially varying flow characteristics, namely (i) advance phase, (ii) wetting or storage phase, (iii) depletion phase and (iv) recession phase. Figure 15 shows a schematic plot of the irrigation times over the furrow distance.

The **advance phase** starts at the time \( t = t_{is} \), i.e. when the irrigation water enters the furrow. During advance, the water moves downstream of the furrow with a characteristic wave front \( x_{surf} \). At the time \( t = t_s \), when the tip of the surface water body reaches the lower field boundary \( x = x_L \), the advance phase ends and the **storage phase** begins. The storage usually consumes the major part of the irrigation time. It is again subdivided into the early and the late storage phase (S-I and S-II) as described in section 3.3.2. Storage ends at the time \( t = t_{co} \) when the inflow discharge is cut off, i.e. when sufficient water has been supplied to the field. Then the **depletion phase** starts and whereas the movement of water continues to progress downstream, the flow depth \( h_{fl} \) at the furrow inlet begins to diminish. At the time \( t = t_r \), when the flow depth at the inlet has approached zero, depletion is completed and the **recession phase** starts. The furrow now dewaters progressively from the upper to the lower field boundary until all water has left the surface by runoff or infiltration at the time \( t = t_{ic} \).
Flow equations
The non-uniform and unsteady surface flow in furrows is usually described by the differential equations of the one-dimensional open-channel flow. These equations are based on the conservation of mass and momentum and are referred to as the complete hydrodynamic (HD) flow equations. Different simplifications of the momentum equation lead to altogether four classes of these equations. The zero-inertia (ZI) approach assumes that all inertial and acceleration terms of the momentum equation can be neglected. Neglecting, furthermore, the depth gradient along the channel leads to the normal-depth or kinematic-wave (KW) equations. Finally, the volume-balance (VB) approach embodies the most severe simplification by ignoring the law of momentum conservation. The choice as to one or other of the approaches for modelling surface flow is dependent upon the required simulation accuracy and other factors such as site specifications and the availability of certain input data.

Surface irrigation models are usually classified according to the above classification of surface flow equations. Except for the most simplistic approaches, e.g. volume balance, the governing flow equations are generally solved numerically at a succession of time or space intervals, where the solution at the end of the time/space step is a direct consequence of the initial values at the beginning of the time step.

Complete hydrodynamic equations
The differential equations for unsteady one-dimensional open-channel flow are the so-called Saint-Venant equations. These equations are based on the continuity equation

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} + q = 0$$

and the unsteady momentum equation which reads in its discharge form:

$$\frac{1}{Ag} \frac{\partial Q}{\partial t} + \frac{2Q}{Ag} \frac{\partial Q}{\partial x} + (1 - F^2) \frac{\partial h_{uf}}{\partial x} = S_a - S_f$$

with
- \(Q\) = the discharge
- \(A\) = the cross-sectional area
- \(t\) = the time

Figure 15 Hydraulic phases of an irrigation event
Methods: the modules of GAIN-P

\[ q = \text{the infiltration rate by unit length} \]
\[ h_{nl} = \text{the flow depth} \]
\[ g = \text{the acceleration of gravity} \]
\[ s_0 = \text{the channel slope} \]
\[ s_f = \text{the friction slope}. \]

The squared Froude number \( F^2 \) is defined by

\[ F^2 = \frac{Q^2 W}{A^3 g} \]  \hspace{1cm} (33)

where \( W \) denotes the top width of the channel. The Froude number \( F \) is also defined by the ratio of the flow velocity to the wave celerity. \( F \) less than 1 indicates sub-critical flow and \( F > 1 \) indicates super-critical flow.

The Saint-Venant equations are commonly used in open channel hydraulics and verified in many literature sources, e.g. Bassett and Fitzsimmons [1976], Katopodes and Strelkoff [1977], Esfandiari and Maheshwari [2001]. Nevertheless, the description of unsteady, gradually varying flow by equations 31 and 32 is based on some assumptions:

- parallel stream lines, i.e. the flow is assumed to be one-dimensional
- the land slope is small so that the sine of the slope angle is approximately the slope itself
- the water is incompressible and has no viscosity, i.e. the friction slope is assumed to be the same as for a uniform flow having similar flow velocity
- the flow channel is prismatic, i.e. the cross-section geometry is invariant along the furrow distance.

These assumptions are necessary for maintaining mathematical simplicity but are often not accurate for surface-irrigated systems. The roughness of the soil surface changes both during a single irrigation event and from one irrigation to the next due to particle transport, weeds, etc. Additionally, the shape and alignment of the furrow cross section may change significantly in irrigated furrows throughout the growing season [Wöhling, 1999], particularly between the first and the second irrigation event. However, it is usually sufficient to admit the existence of these effects unless they have a substantial impact on the modelling accuracy.

Zero-inertia equations

The hydrodynamic model is often too expensive to operate and may be difficult to utilize because of its complexity [Walker and Skogerboe, 1987]. Its often laborious numerics can also cause instability. In order to overcome these limitations, various simplifications of the HD equations have been introduced. The zero-inertia assumption that the inertial and acceleration terms in the momentum equation 32 are negligible leads to:

\[ \frac{\partial y}{\partial x} = S_0 - S_f \]  \hspace{1cm} (34)

which, together with the unchanged continuity equation 31, is referred to as the ZI model.
Kinematic wave equations
If the bottom slope of the channel is sufficiently steep, the depth gradient of Equation 34 is much smaller than either of the right-hand terms, which are then more or less essentially balanced [Bassett and Fangmeier, 1980]. Thus,

\[ S_f = S_0 \quad (35) \]

holds true at any point along the furrow and the flow is at normal depth. Any prescribed relation between depth or area and discharge (e.g. uniform flow equations by Manning, Chezy or Darzy-Weisbach) coupled to a continuity equation such as Equation 31 are referred to as the 'KW model'.

Volume balance approach
The VB approach completely neglects the momentum equation 32 and replaces the water flow dynamics by gross assumptions. A steady discharge at the field inlet \( Q_0 \) is compulsory for this approach. When setting up this condition, the product of \( Q_0 \) and the time \( t \) must equal at all times the volume of water on the soil surface \( V_{surf} \) plus the infiltrated volume \( V_{inf} \) which are both time-dependent. The equation

\[ Q_0 t = V_{surf}(t) + V_{inf}(t) \quad (36) \]

is equivalent to the integrated form of Equation 32. Without the momentum equation, the shape of the surface water profile and the volume of surface storage \( V_{surf} \) is unknown. This was realized by Lewis and Milne [1938] who assumed an averaged flow depth in the channel, constant in time and space, and formulated the equation which is the basis of most volume-balance models:

\[ Q_0 t = \bar{A} x_{tip} + \int_0^{x_{tip}} I_{inf}(t - t_x) \, dx \quad (37) \]

with
\[ \bar{A} \]
= the average flow area
\[ x_{tip} \]
= the advance length
\[ I_{inf}(x,t) \]
= the infiltrated volume per unit length
\[ \tau = t - t_x \]
= the infiltration opportunity time.

The advancing tip of the surface water body reaches the distance \( x \) at time \( t_x \). The volume-balance approach is applied primarily to the advance phase, although it has also been developed for the other hydraulic phases, too.

Initial and boundary conditions
The surface flow equations describe relationships between the time and distance rates of change of the unknown variables, namely, depth and discharge. Before a solution for the actual values of these quantities at various times and locations can be determined, their initial state must be specified, as well as their values at the two ends (boundaries) of the surface water body. Therefore, the solution develops firstly in response to the equations governing the flow and secondly to the initial and boundary conditions [Bassett and Fangmeier, 1980].
Methods: the modules of GAIN-P

Initial conditions
Initial flow depth and discharge are necessary for the solution of the governing equations 31 and 32 at the first time step. Both quantities are initially zero since the irrigation is usually conducted on empty furrows. Numerical solution techniques are not able to start calculations with these initial conditions. The sudden rise in flow depth and discharge is too much for most, if not all, numerical solution techniques. In order to overcome this limitation, an assumption regarding the shape of the surface water body (e.g. a power-law shape) is constituted for a first small time increment. Bassett and Fangmeier [1980] show that this assumption allows the determination of both the location of the downstream boundary and the flow depth at the furrow inlet. According to Bassett and Fangmeier [1980] and Schmitz and Seus [1987], it was found that although the results of the first time step are erroneous, the error rapidly diminishes after a few time increments.

Boundary conditions
On the occasions when the furrow is completely filled with water (during storage phase), the upper and lower boundary of the surface flow are the furrow inlet and the outlet, respectively. At these times, the upstream boundary condition is the known inflow discharge $Q_i$ and the downstream boundary condition is the discharge at the furrow outlet $Q_L$. The latter is either zero, if the downstream boundary has been blocked to prevent runoff, or a free-overfall condition, i.e. $Q_L$ is a known function of the flow depth $h_{it}$.

However, during both the irrigation advance and recession phases, the field is only partly covered with water. At least one boundary is unknown, which makes the modelling of the surface flow more complex as compared to the modelling of the flow during the storage phase. Both the flow depth and discharge at the advancing wave tip is zero, but the velocity is not. Moreover, the location, shape and velocity of the irrigation advance on dry soil cannot be defined a priori [Schmitz, 1989]. A specific shape of the wave front (e.g. parabolic shape) is often assumed in order to have at least one feasible boundary. The coefficients of the shape function are determined using a continuity approach, which at least satisfies the volume balance requirements.

During the recession phase, a trailing edge of surface water body moves down the furrow. It constitutes the location of the upstream boundary where, again, both flow depth and discharge are zero. The shape of the receding wave tip, however, is different from the shape of the advancing tip. Usually it can be characterized by a horizontal water level.

3.3.1.3 Subsurface flow
With respect to the water supply of the plants, the water storage within the root zone is a highly dynamic process. A further difficulty for realistically portraying the related water transport processes arises from the spatially and even temporally varying soil characteristics, e.g. the swelling and shrinking of clay. With respect to irrigated agriculture, relevant soil characteristics are: (i) the capacity of the soil to hold water and yet still be well drained, (ii) the water flow properties in the soil, and (iii) the physical properties of the soil matrix, e.g. the organic matter content, soil depth, soil texture and soil structure [Walker and Skogerboe, 1987].

The important question Which fraction of the irrigation water becomes available for the plant roots at which time and at which location? can only be answered by analyzing the movement of the infiltrated water within the soil. This section thus deals with the principles for
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calculating the subsurface-flow dynamics. It begins with the characterization of the terms 'infiltration' and 'redistribution'.

**Infiltration and redistribution**

**Infiltration** is defined as "the process of the entry into the soil of water made available at its surface" [Philip, 1969]. The infiltration rate at a point of the furrow is defined as "the volume flux of water flowing into the soil profile per unit of surface area". It is also a term of the surface flow equations, namely, the continuity equation 31. In furrow irrigation, the water available on the surface often exceeds the soil's infiltration capacity and, thus, the maximum infiltration rate. This rate refers to atmospheric pressure and can only be exceeded if the rising water level in the furrow imposes a hydrostatic pressure on the soil surface\(^1\). Infiltrability and its variation with time is known to depend on the initial soil wetness and suction head, as well as on the texture, the structure and the uniformity of the soil profile. Hillel [1998] summarizes five main factors on which the infiltration rate depends:

- Infiltration opportunity time. The infiltration rate is relatively high at initial wetting, it decreases gradually and eventually approaches asymptotically to a constant rate that is characteristic for the soil under consideration.
- Initial water content. The wetter the soil is initially, the lower the initial infiltration rate.
- Hydraulic conductivity. A higher saturated hydraulic conductivity is generally associated with a higher infiltration rate.
- Surface conditions. The compaction of the soil surface zone lowers the infiltration rate. The infiltrability is smaller for uniform soil than for a highly porous soil surface with 'open' structure.
- Profile depth and layering. Fine-textured (clayey) soils impede the water flow due to their lower hydraulic conductivity. The infiltration is initially retarded when entering dry, coarse-textured (sandy) soils due to a reduced capillary tension.

It is shown that the soil type can have a controlling (limiting) effect on the infiltration rate either at the surface or within the profile. In contrast to border irrigation, the flow depth has a significant impact on the infiltration rate from furrows. The wetted furrow perimeter increases with flow depth at a rate which depends on the furrow geometry. Along with the wetted perimeter, the infiltration rate increases likewise with flow depth. Field experiments have revealed that the infiltration rate from furrows is greater than the one from borders, even when the width of the border is equal to that of the wetted furrow perimeter [Philip, 1984; Schmitz et al., 1991].

Figure 16 shows a schematic plot of the wetting front in the soil below an irrigated furrow \(A_f\) and below an irrigated border \(A_b\), both having the same wetted surface length. At short infiltration opportunity times, i.e. soon after wetting, the area of water-saturated soil below both the furrow and the border are almost equal in size. For longer opportunity times, however, the saturated area below the furrow becomes progressively larger than the saturated area below the border: \(A_f = \Lambda \cdot A_b\). For a given homogeneous soil, the rate of increase \(\Lambda(t)\) depends on both infiltration opportunity time and furrow curvature.

\(^1\) The average flow depth in an irrigated furrow is usually well below 0.05 m. This depth would cause the pressure at the soil surface to rise by as little as 5% or less. As a result, the infiltrability can be assumed to be the maximum infiltration rate.
Methods: the modules of GAIN-P

Infiltration intake formula – In most furrow irrigation models, infiltration is merely considered as a sink term which is only dependent on infiltration opportunity time $\tau$ and some empirical parameters. The Kostiakov-Lewis formula, based on Kostiakov [1932], is frequently used:

$$I_k(\tau) = k_k \cdot \tau^{a_k} + c_k \cdot \tau$$

(38)

with

$I_k = \text{the cumulative infiltration per unit length}$

$a_k, k_k = \text{the empirical Kostiakov parameters}$

$c_k = \text{the basic intake rate}$.

The corresponding formula to calculate the infiltration rate per unit length reads:

$$q_k(\tau) = k_k a_k \tau^{a_k-1} + c_k.$$ 

(39)

The Kostiakov parameters are not universally applicable and only account for the conditions of the irrigation system for which they are calibrated, e.g. the initial soil conditions, the flow depth, etc. If the initial and boundary conditions of the irrigation system differ from those of the calibration, the parameters must be re-calibrated.

Redistribution – The process of infiltration comes to an end when the free water at the soil surface disappears. Water movement within the soil, however, may persist for a long time afterwards as the soil moisture percolates within the profile. This movement is called redistribution, as long as the soil profile has not been initially wetted to saturation throughout its depth. Redistribution often determines how much water flows through the root zone, rather than being retained within it. The soil-water storage within the root zone is generally not a fixed quantity nor a static property but a dynamic process, which is determined by the time-variable rates of soil-water inflow to, and outflow from, this zone. The redistribution process necessary involves hysteresis, since the relationship between the water content and the suction is not unique but depends on the history of wetting and drying in the soil profile [Hillel, 1998].
Dynamics of water flow

Empirical infiltration formulae like the Kostiakov equation are only applicable for calibrated conditions. They do not consider the varying flow depth neither at a cross section, nor along the furrow reach. Moreover, nothing can be stated about the subsurface water transport nor about the soil moisture distribution. Physical models, on the other hand, describe the dynamics of water flow in the saturated-unsaturated soil by a certain formulation of the two-dimensional Richards equation [Richards, 1931]. These models employ physical parameters which do not have to be calibrated; they consider the actual (observed) initial conditions and allow for the simulation of water transport in a wide range of soils. The two-dimensional infiltration from arbitrarily shaped channel cross sections is correctly taken into account by considering the local flow-depth hydrograph. Although soil-parameter estimation is sometimes difficult, these models can be used to calculate the transient water distribution in most irrigated soils.

In this section, the fundamentals for modelling the water flow dynamics are taken from technical literature: the water potential concept, the Richards equation and the functions for describing the soil hydraulic properties. For more detailed information on this topic we recommend consulting the literature on soil physics by Simúnek et al. [1996], Hillel [1998] and Flühler and Roth [2003].

Soil water potential concept

The water potential $\psi_w$ is the total potential energy of the soil water related to the mass of water [J kg$^{-1}$]. It is the sum of gravitation potential $\psi_g$, pressure potential $\psi_p$ and osmotic potential $\psi_o$:

$$\psi_w = \psi_g + \psi_p + \psi_o \quad (40)$$

Its slope, i.e. the negative hydraulic gradient $\partial \psi_w / \partial z$, is the driving force for water movement in the soil. In unsaturated soils, the pressure potential equals the suction of the soil matrix, i.e. the matric potential $\psi_m$. The presence of solutes in the soil water affects the thermodynamic properties of the water and lowers its potential energy. However, these effects generally play a minor role as regards water transport processes and thus can be neglected, which leads to:

$$\psi_w = \psi_g + \psi_p \quad (41)$$

If the energy of the soil water is related to the weight of unit mass $gM_w$ instead of only to mass, the hydraulic head $h_w [m]$ is defined as

$$h_w = -z + h_p \quad (42)$$

where $z$ denotes the depth related to the reference vertical position $z_0 = 0$ (i.e. soil surface) and $h_p$ is the pressure equivalent in the soil related to atmospheric pressure$^2$. The hydraulic head $h_w$, in its simplest case, has two components originating from gravity and soil capillarity:

---

$^2$ Equation 42 accounts for rigid non-deformable soils, where both air and liquid phases are continuously interconnected, where soil air is at atmospheric pressure and osmotic energy is negligible [Flühler and Roth, 2003].
\[ h_w = -z + h_m \]  

(43)

where \( h_m \) denotes the matric head or capillary height \( h_w = \psi_m / (\rho_v g) \).

**Richards equation**

The soil water potential concept is a fundamental base for describing the water flow in the soil. Richards [1931] combined the volume balance equation with the Buckingham-Darcy law for unsaturated flow

\[ q_w = -K(\theta_w) \frac{\partial}{\partial z} h_w \]  

(44)

with

\[
q_w = \text{the volumetric water flow (flux)}
\]

\[
K = \text{the hydraulic conductivity}
\]

\[
\theta_w = \text{the volumetric water content}.
\]

The resulting (one-dimensional) equation:

\[
\frac{\partial}{\partial t} \theta_w - \frac{\partial}{\partial z} \left( K(\theta_w) \frac{\partial}{\partial z} h_w \right) = 0
\]

(45)

has a clear physical basis. It can be used for fundamental research and scenario analysis [Dam and Feddes, 2000] and is employed in most physical soil-water transport models. The water flow in water-saturated soils is described by a special case of the Richards equation, where the volumetric water content is constant in time and is equal to porosity. In this case, the hydraulic conductivity is equal to the saturated hydraulic conductivity \( K_s \).

Although the Richards equation is applied almost universally for describing soil-water transport in saturated-unsaturated soils, it has a limited area of validity. Flühler and Roth [2003] summarize the limitations:

- The flow equation 44 is based on the assumption that \( q_w \) is proportional to the driving force \(- \partial h_w / \partial z\). Deviations from this linear relationship may occur due to turbulent flow in coarse textured soils and due to water adsorption in fine textured soils.
- \( \theta_w \) and \( h_w \) are defined for describing equilibrium conditions in a soil volume. The water potential reacts faster than the water content to a sudden disturbance of the equilibrium, e.g. irrigation or rainfall events. The flow equation 44 is therefore only valid, if \( \theta_w \) and \( h_w \) do not significantly diverge from the equilibrium. Consequently, the faster the considered phenomena, the smaller must be the domain where Equation 44 is applied.
- Air can be trapped in the pores when water penetrates from the soil surface. The trapped air pressure is no longer equal to atmospheric pressure and must be considered as an additional component of \( h_w \).
- The description of water transport in deformable soils is especially difficult. The hydraulic properties of the soil may be variable when the soil matrix, i.e. the pore space, is not rigid.

---

\(^3\) \( h_p \) is replaced by \( h_m \) in Equation 42 for unsaturated soils.

\(^4\) For example, HYDRUS-2D utilizes the two-dimensional Richards equation as described later on in the text.
Finally, the soil water can also be transported in its vapour phase. This phenomena is especially relevant in the arid climate zone.

The use of the Richards equation is often the only (accessible) possibility for calculating the water transport in variably saturated soils. However, before using the Richards equation, careful consideration needs to be given as to whether the necessary assumptions hold true or not for the soil under consideration.

Soil hydraulic properties

The relation between volumetric water content in the soil \( \theta_w \) and matric head \( h_m \) (or matric potential \( \psi_m \)) is a function of the pore space geometry as well as the physical and chemical properties of the soil. This relationship is generally highly nonlinear and often distinctly hysteretic. In the present contribution, the soil-hydraulic model proposed by van Genuchten [1980] is used in its modified form by Vogel et al. [2001] to calculate the functions \( \theta_w(h_m) \) and \( K(h_m) \):

\[
\theta_w(h_m) = \begin{cases} \theta_r + \frac{\theta_m - \theta_r}{1 + \left| \alpha h_m \right|^n}, & h_m < h_s \\ \theta_s, & h_m \geq h_s \end{cases}
\]  

(46)

\[
K(h_m) = \begin{cases} K_r(h_m), & h_m < h \\ K_s, & h_m \geq h_s \end{cases}
\]  

(47)

where

\[
K_r(S_e) = S_e^{0.5} \left[ \frac{1 - F(S_e)}{1 - F(1)} \right]^2
\]  

(48)

\[
F(S_e) = \left[ 1 - S_e^{1/m} \right]^m
\]  

(49)

\[
S_e^* = \frac{\theta_1 - \theta_r}{\theta_m - \theta_r} S_e
\]  

(50)

with

- \( n, \alpha \) = the empirical shape parameters
- \( m \) = \( 1 - 1/n \)
- \( \theta_s \) = the volumetric water content at saturation
- \( \theta_r \) = the residual water content
- \( \theta_m \) = a fictitious (extrapolated) parameter (\( \theta_m \geq \theta_r \); Vogel et al., 2001)
- \( h_s \) = the minimum capillary height, at which a continuous non-wetting phase exists (\( h_s < 0 \)).

The characteristic curves of the \( \theta_w(h_m) \) and \( K(h_m) \) functions of three different soils are given in Figure 17.
3.3.1.4 Crop growth

The development of agricultural crops depends on a wide range of environmental factors such as the intensity, the duration and the spectral distribution of solar radiation, the air temperature, the wind speed, the soil water content and the chemicals in both the soil and the air. Detailed studies of plant composition and transpiration lead to the conclusion that about 95%, or even more, of the water extracted from the soil by plant roots is transpired and flows through the soil-root-stem-leaves-atmosphere system [Kutilek and Nielsen, 1994]. The resulting concept of a soil-plant-atmosphere continuum describes the flux through the above system with an analogy to the current in an electrical resistance network (cf. e.g. Oke [1987], Monteith and Unsworth [1990], Larcher [1994], Kutilek and Nielsen [1994]). Even more simplified equations are obtained by assuming that transpiration is equal to the water uptake by plants. Although there is a time shift (diurnal variation) between the two processes, this is a reasonable assumption for daily averages of transpiration.

Irrigation is conducted primarily in order to meet the crop water requirements at times and/or in areas where the natural rainfall is too low. Crop water requirements are defined as the "…water needed to meet the water loss through evapotranspiration of a disease-free crop, growing on large fields under non-restricting soil conditions ... and achieving full production potential under a given growing environment" [Doorenbos and Pruitt, 1992].

Evapotranspiration

The composite loss of water to the air from evaporation and transpiration is called evapotranspiration (ET) [Oke, 1987]. Evaporation and transpiration occur simultaneously and there is no easy way of distinguishing between the two processes. Apart from the water availability in the topsoil, the evaporation from a cropped soil is mainly determined by the fraction of the solar radiation reaching the soil surface. This fraction decreases over the growing period as the crop develops and the crop canopy shades more and more of the ground area. When the crop is small, water is predominately lost by soil evaporation (Figure 18), but
once the crop is well developed and completely covers the soil, transpiration becomes the dominant process [Allen et al., 1998].

Transpiration and evaporation are described mathematically by the flux averaged over a representative volume of roots and the upward water flux across the soil-surface respectively. These approaches are consistent in the macroscopic scale similar to the description of soil-water transport by the Richards equation. Although terminology is ambiguous in literature, the term 'potential evapotranspiration' (ETP) is defined here by an unlimited water supply to both the soil surface and the vegetation. The evapotranspiration rate from a hypothetical short grass crop with unlimited water supply is called 'reference evapotranspiration' \( ET_0 \). It expresses the evaporating power of the atmosphere at a specific location and time of the year and does not consider crop characteristics and soil factors. Thus, \( ET_0 \) is affected by climatic parameters only and can be computed from weather data. As a result of an expert consultation held in May 1990, the physically based FAO Penman-Monteith method [Penman, 1948; Monteith, 1965] is now recommended as the sole standard method for the definition and computation of the reference evapotranspiration [Allen et al., 1998]. The FAO Penman-Monteith method requires radiation, air temperature, air humidity and wind speed data. Empirical methods to estimate \( ET_0 \) such as the ones by Haude [1958], Turc [1961] and Blaney and Criddle [1950] should only be used if their parameters are calibrated on the Penman-Monteith equation.

The actual crop evapotranspiration \( ET_{c,a} \) depends on the climatic conditions, the crop variety and the stage of growth, the available soil moisture in the root zone, and the various physical and chemical properties of the soil.

### 3.3.1.5 Irrigation performance criteria

For theoretical aspects, irrigation performance criteria are useful for the modeller when it comes to analyzing the performance of rather complex model systems. In practice, users of irrigation water often have to defend their share of the water resource with the argument that it is necessary and wisely used [Burt et al., 1997]. For them it is also important to quantify the performance of their irrigation systems.

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5 A reference grass crop with an assumed crop height of 0.12 m, a fixed surface resistance of 70 s m\(^{-1}\) and an albedo of 0.23. The reference crop closely resembles an extensive surface of green, well-watered grass of uniform height, actively growing and completely shading the ground.
During the past decades, various definitions of performance indicators, usually called 'efficiencies', have been given in literature. To clarify terms and to establish standards, the On-farm Irrigation Committee of the Irrigation and Drainage Division, American Society of Civil Engineers (ASCE) provided a principle work on irrigation measures, which was updated by the ASCE Task Committee on Defining Irrigation Efficiency and Uniformity [Burt et al., 1997]. The definitions in Burt et al. [1997] aim at being consistent regardless of the region under consideration. However, in this subsection they are only adopted for the application to the field level (or to a single furrow). Four performance measures are subsequently presented, namely the irrigation efficiency (IE), the application efficiency (AE), the distribution uniformity (DU) and the adequacy (AD).

**Irrigation efficiency**
The irrigation efficiency $IE$ is commonly defined as the ratio between the volume of irrigation water beneficially used and the total volume of irrigation water actually applied to the field [Heermann et al., 1990; Clemmens and Dedrick, 1994]. A more general definition is proposed by Burt et al. [1997] which reads:

$$IE = \frac{V_b}{V_{in} - \Delta S_w} \times 100\%$$  \hspace{1cm} (51)

with:

- $V_{in}$ = the volume of applied irrigation water
- $\Delta S_w$ = the change in irrigation water storage
- $V_b$ = the volume of irrigation water beneficially used 6.

Beneficial uses of irrigation water are the crop evapotranspiration $ET_{c,a}$, the water harvested with the crop, the salt removal, the soil preparation, etc. The evaporation is a part of $ET_{c,a}$ and is included in the beneficial uses (because of the practical inability to quantify just how much evaporation is unavoidable) rather than limiting $ET$ to something closely resembling transpiration of irrigation water [Burt et al., 1997]. The denominator in Equation 51 represents the total volume of irrigation water that leaves the domain boundaries (outflow = inflow – $\Delta$ storage) within a specific time interval.

Irrigation efficiency can be approximately estimated for a single irrigation (specifying the time interval from just before an irrigation event to just before the next irrigation) or a rough estimate is also feasible for the entire growing season. Technically, Equation 51 represents the ratio between two volumes leaving the system during a specific time interval: (i) the beneficially used irrigation water (numerator) and (ii) the irrigation water that left the system for beneficial plus non-beneficial uses (denominator) respectively. The IE has some significance in distributed modelling, but less significance for practical purposes because the terms of Equation 51 are very difficult to evaluate in the field.

**Application efficiency**
The application efficiency $AE$ is based on the concept of meeting a target irrigation depth or volume by the water application (irrigation). In the context of the present study, $AE$ is the

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6 The phrase 'irrigation water' excludes water applied naturally to the crop, like precipitation.
volume of irrigation water contributing to the target $V_t$, divided by the volume of irrigation water applied to the field:

$$AE = \frac{V_t}{V_{in}} \times 100\%$$  \hspace{1cm} (52)

The chosen target may be the soil moisture deficit (SMD) in the root zone or some smaller amount. In theory, the application efficiency can be 100% if all the water delivered is available for beneficial uses. This, however, is hardly achievable under field conditions.

**Distribution uniformity**

The distribution uniformity (DU) is not an efficiency term but a measure of the uniform manner in which irrigation water is distributed to different areas in a field. It is usually defined as a ratio of some measure of the smallest accumulated volume in a fraction of the field, compared to the average accumulated volume in the entire field. It has proven to be practical and useful in irrigated agriculture to divide the furrow length into four equidistant parts. This leads to the low-quarter distribution uniformity $DU_{lq}$, which is defined as:

$$DU_{lq} = \frac{d_{lq}}{d_{av}}$$  \hspace{1cm} (53)

with

- $d_{lq}$ = the volume infiltrated in the 1/4 total surface area with the smallest infiltrated volume (usually the most downstream quarter of the furrow), divided by 1/4 of the total surface area
- $d_{av}$ = the average volume of water infiltrated in the flow domain, which is the total infiltrated volume in all elements, divided by the total surface area.

**Adequacy**

Adequacy is a parameter complementary to AE, indicating the degree in which the target or required volume $d_{req}$ is met and it should be included in any list of pertinent performance measures [Burt et al., 1997]. For practical reasons, the portion of the field with the least infiltrated volume is considered by the low-quarter adequacy $AD_{lq}$, which is defined as

$$AD_{lq} = \frac{d_{lq}}{d_{req}}$$  \hspace{1cm} (54)

If the average low-quarter volume $d_{lq}$ is used as the scheduling criterion, then a proper irrigation duration is chosen, where $AD_{lq} = 1.0$, with about 1/8 of the furrow remaining under-irrigated. With this definition, $AD_{lq} < 1.0$ implies under-irrigation, whereas $AD_{lq} > 1.0$ implies over-irrigation [Burt et al., 1997]. The average required volume $d_{req}$ depicts the average volume of water required to compensate the soil moisture deficit of the total root zone, completely or to some extent, along the furrow.
Evaluation of performance criteria

The potential performance of an irrigation system depends on both monetary and non-monetary factors. These include the water costs and availability, the labour costs, the crop type and its resistance against environmental stress, the water rights, the social aspects, the irrigation tradition and the management strategies (e.g. a two-weekly water turn in Pakistan, called warabandi). The performance of an irrigation system may be optimal when the moisture level is suitably maintained, and the evaporative, runoff and percolation losses are minimized\(^7\). In order to evaluate the system's performance, several criteria are used, which are based solely on the irrigation-water balance. But until now, no one single parameter exists which is sufficient for evaluating irrigation performance [Walker and Skogerboe, 1987]. A reasonable minimum of terms, taken together, can yield useful information suitable for decision-making. Several examples are subsequently given to illustrate the need for a performance-criteria cocktail.

Irrigation efficiency is difficult to measure in the field and does not take the uniformity of the water application into account. A high percentage of irrigation efficiency does not necessarily result in optimal crop yield because the water may be distributed non-uniformly along the furrow reach. On the other hand, however, a high value of distribution uniformity does not necessarily mean that the water becomes completely accessible for the plant roots because this measure states nothing about possible losses by evaporation and deep percolation. Furthermore, it is even possible to attain a very high application efficiency (AE) percentage in a field which is nevertheless under-irrigated. In this case, the adequacy parameter (AD) is additionally needed to indicate the degree in which a required water application is met.

In this study, altogether four performance criteria \((IE, AE, DU, AD_\text{AD})\) are used in order to get a comprehensive picture of the irrigation system performance.

### 3.3.2 Surface flow model

A closed-form solution for the description of surface flow in irrigated furrows is difficult to develop due to the time and space-dependent character of flow during the various phases of the irrigation. Furthermore, a closed-form solution is numerically not very efficient either. In furrow irrigation modelling, different approaches are used for the simulation of the different irrigation phases.

The surface flow model FAP, which is presented subsequently, is designed to model surface irrigation on a single, gently sloped and prismatic channel of arbitrary cross section during all the phases of irrigation. Moreover, any given infiltration function, empirical, conceptional or physically based, can be coupled with the model. FAP integrates three different surface flow models for (i) the advance and the early storage phases, (ii) the late storage phase, and (iii) both the depletion and the recession phases. The mathematical derivation of these individual models, together with their initial and boundary conditions, are presented in full detail in the following subsections.

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\(^7\) A certain quantity of surface runoff and/or deep percolation, however, may be required for salt removal.
3.3.2.1 Advance phase model

Schmitz [1989], Schmitz and Seus [1990] developed an analytical ZI model for both the advance and the storage phases. The concept, which combines the two hydraulic phases, is the so-called concept of the 'virtual wave'. It comprises the association that the field is extended virtually beyond its real dimension so far that the irrigation advance never reaches the lower boundary during the entire simulation. By simulating the irrigation advance in the virtual field, both the advance and the storage phases within the real field dimensions are covered as seen in Figure 19. During the storage phase, the virtual wave provides, apart from the flow depth along the furrow \( h_{\text{wf}}(x, t) \), a good approximation of the time-variable flow rate at the real field outlet \( Q_{L}(t) \) (Figure 19).

![Figure 19 Modelling irrigation advance – the concept of the ‘virtual wave’](image)

For simulating the advance (and early storage) phase, FAP utilizes the concept of the virtual wave, which was adopted for furrow irrigation by Schmitz and Seus [1992]. The analytical ZI equations are re-derived and subsequently given in full detail because they differ slightly from the equations found by Schmitz and Seus [1992]. But like the model by Schmitz and Seus [1992], the new model involves all the common types of furrow cross sections and takes into account the varying, in time and space, character of infiltration by a sink term \(-q\). Thus, any infiltration function can be implemented in the advance flow model. Since the influence of the infiltration process on momentum balance is, contrary to ZI tradition, taken into account, the ZI equations for a prismatic furrow of arbitrary cross section read [Schmitz and Seus, 1992]:

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = -q
\]

(55)

\[
\frac{\partial h_{w}}{\partial x} = S_0 - \frac{v^2}{K_{st}^2 R^4} + \frac{q \cdot v}{g \cdot A}
\]

(56)

with

- \( h_{w}(x, t) \) = the flow depth
- \( v(x, t) \) = the flow velocity
- \( R \) = the hydraulic radius
- \( K_{st} = 1/n \) = the roughness coefficient, where \( n \) = the Manning's coefficient.
Methods: the modules of GAIN-P

Multiplication of Equation 56 with the factor $\hat{r}^2$ and substitution of $H_{nl} : (x,t) \mapsto H_{nl}(x,t)$ parameterized form $\tilde{h}_{nl} : A(x,t) \mapsto \tilde{h}_{nl}[A(x,t)]$ yields:

$$R^2 \frac{\partial A}{\partial x} \frac{\partial \tilde{h}_{nl}}{\partial A} = - \frac{v^2}{K_{st}} + (S_0 + \frac{q \cdot v}{g \cdot A}) R^2$$

In Equation 57 the variables $A$, $q$ and $v$ are dependent on time and space. In order to reduce the dependency on time alone, the momentum described by the right-hand side of Equation 57 is replaced by the momentum of the moving centre of gravity $\bar{x}$ of the surface water body (cf. Figure 20). Thus, Equation 57 changes to [Schmitz and Seus, 1992]:

$$R^2 \frac{\partial A}{\partial x} \frac{\partial \tilde{h}_{nl}}{\partial A} = - \frac{\bar{v}^2}{K_{st}} + (S_0 + \frac{\bar{q} \cdot \bar{v}}{g \cdot A}) R^4$$

where $\bar{q}(t) = q[\bar{x}(t), t]$; $\bar{A}(t) = A[\bar{x}(t), t]$; $\bar{R}(t) = R[\bar{x}(t), t]$ and $\bar{v}(t) = v[\bar{x}(t), t]$ are the infiltration rate, the cross-sectional area, the hydraulic radius and the flow velocity at $\bar{x}$, respectively. The initial and boundary conditions of the differential equations 55 and 4.41 are:

$$x_{ip}(0) = 0$$

$$Q(0, t) = Q_0(t)$$

$$A[x_{ip}(t), t] = 0$$

$$v[x_{ip}(t), t] = v_{ip}(t)$$

where $x_{ip}$ and $v_{ip}$ denote the location and velocity of the advancing water front (cf. Figure 20).

---

Figure 20  Principles of modelling furrow irrigation advance
Irrigation control: towards a new solution of an old problem

The Equation (9) in Schmitz and Seus [1992] can be utilized for prismatic furrows and is inserted into Equation 58. Using the abbreviation

$$\bar{\rho}(t) = -\frac{V^2}{K_s} + \left( S_0 + \frac{q \cdot V}{g \cdot A} \right) R^4$$

(63)

the derivation of the analytical solution of Equation 58 is subsequently outlined:

$$c_s A(x,t)^{\gamma_s} \frac{\partial A}{\partial x} = \bar{\rho}(t)$$

$$\frac{\partial}{\partial x} A(x,t)^{\gamma_s+1} = \frac{\gamma_s+1}{c_s} \bar{\rho}(t)$$

$$A(x_{tip},t)^{\gamma_s+1} - A(x_{tip},t)^{\gamma_s+1} = \frac{\gamma_s+1}{c_s} \int_x^t \bar{\rho}(t) d\xi$$

Finally, the analytical solution reads:

$$A(x,t) = A_0(t)^{\gamma_s+1} \left( 1 - \frac{x}{x_{tip}(t)} \right)$$

(65)

where $\gamma_s$, $c_s$ and $\alpha_s$ denote geometric coefficients of the furrow cross section. The coefficients for the general furrow shape functions are:

$$\alpha_s = -\frac{1}{\gamma_s+1}$$

(66)

$$\gamma_s = 4 \frac{p_1}{3} + p_2 - 1$$

(67)

$$c_s = p_1 \cdot p_2 \cdot p_3^{(4/3)}$$

(68)

where $p_1$, $p_2$, $p_3$, $p_4$ are parameters of the hydraulic section:

$$h_{wf}(A) = p_1 A^{p_2}$$

(69)

$$R(A) = p_3 A^{p_4}$$

(70)

For parameters of the triangular and parabolic cross sections see Schmitz and Seus [1992]. The integration of Equation 55 with consideration of the boundary condition Equation 60, analogous to the methods in Schmitz and Seus [1992], leads to the formula:
Methods: the modules of GAIN-P

\[ Q(x,t) = Q_0(t) - \int_0^{x_{tip}} \frac{\partial A}{\partial t}(\hat{x},t)d\hat{x} - \int_0^{x_{tip}} q(\hat{x},t)d\hat{x} \]  
\[ = \frac{\partial V_{inf}}{\partial t}(x,t) \]  

The volume balance of the system is considered to implement the boundary condition Equation 62:

\[ V_{in}(t) = \int_0^{x_{tip}} A(\hat{x},t)d\hat{x} + V_{inf}(x_{tip}(t),t) \]  
\[ (72) \]

Equation 72 is differentiated with respect to time

\[ \frac{\partial V_{inf}}{\partial t}(t) = Q_0(t) = \int_0^{x_{tip}} \frac{\partial A}{\partial t}(\hat{x},t)d\hat{x} + \frac{\partial V_{inf}}{\partial t}(x_{tip}(t),t) \]  
and inserted into Equation 71:

\[ Q(x,t) = \int_0^{x_{tip}} \frac{\partial A}{\partial t}(\hat{x},t)d\hat{x} + \left( \frac{\partial V_{inf}}{\partial t}(x_{tip}(t),t) - \frac{\partial V_{inf}}{\partial t}(x,t) \right) \]  
\[ = \int_0^{x_{tip}} q(\hat{x},t)d\hat{x} \]  

Dividing Equation 73 by Equation 65 yields the following formula for the flow velocity:

\[ v(x,t) = \frac{Q(x,t)}{A(x,t)} = \frac{\int_0^{x_{tip}} \frac{\partial A}{\partial t}(\hat{x},t)d\hat{x} + \int_0^{x_{tip}} q(\hat{x},t)d\hat{x}}{A(x,t)} \]  
\[ (74) \]

Following the first steps of the derivation which leads to Equation 65, the subsequent form of the velocity equation is obtained, which satisfies the boundary condition 62:

\[ v(x,t) = \left( 1 - \frac{x}{x_{tip}(t)} \right) \left( v_0(t) - v_{tip}(t) - \frac{\int_0^{x_{tip}} q(\hat{x},t)d\hat{x}}{A_0(t)} \right) + v_{tip}(t) + \frac{\int_0^{x_{tip}} q(\hat{x},t)d\hat{x}}{A(x,t)} \]  
\[ (75) \]

At this point, the derivation of the analytical solution of the momentum equation is completed. Equation 65, together with the volume balance Equation 71, constitutes the governing equations for the analytical ZI model. In order to calculate the wetted cross-sectional area of the furrow \( A(x,t) \) and the discharge \( Q(x,t) \) for times \( t \) from Equations 65 and 73, values of \( A_0(t) \) and \( x_{tip}(t) \) are required. These values are determined by an iterative scheme by reformulating Equations 65 and 71 as fixpoint equations for \( A_0(t) \) and \( x_{tip}(t) \). Since the variables of the fixpoint equations are time-dependent, the scheme is based on a given time discretization of equidistant time steps \( \Delta t \). The final implicit relation for \( A_0(t) \) and \( x_{tip}(t) \) is different from that of Schmitz and Seus [1992] and reads:
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\[ A_0^{(k+1)}(t) = \left\{ \left[ \frac{\bar{v}(t) A_0^{(k)}(t)}{\kappa_{v,t}} \right] - \frac{\bar{q}(t) A_0^{(k)}(t) R^{(k)^{1/3}}}{8} \left( \frac{a_{z+1}}{a_z} \right) \right\}^{-1/2} \]  

(76)

with \( \bar{v}(t) \), \( \bar{q}(t) \) and \( \bar{R}(t) \) being values of \( v \), \( q \) and \( R \) at the centre of gravity of the surface water body [Schmitz and Seus 1992] and

\[ x_{tip}^{(k+1)}(t) = (\alpha_z + 1) \frac{V_{in}(t) - V_{inf}(t)}{A_0^{(k+1)}(t)} \]  

(77)

Subsequent iterations are indicated by the counter \( k \). The iterative scheme and its numerical behaviour is described in detail in subsection 3.3.5.2.

3.3.2.2 Storage phase model

Utilizing the virtual-wave concept during the entire length of the storage phase, as proposed by Schmitz and Seus [1992], may lead to the phenomenon of the 'standing wave' as follows.

The rate of advance in the (virtual) furrow decreases with time and distance since the area of the soil surface, which participates in infiltration, is on the increase. After a certain advance time, the infiltration rate from the surface water body becomes equal to the inflow rate and the advance rate becomes zero. The zero-advance condition, however, is not defined by the advance model and leads to computer-terminated abortion. As shown in subsection 3.3.5.2, low irrigation advance rates inevitably lead to numerical instability, a decrease in accuracy and an increase in computation time.

In the present study, the storage-phase modelling is therefore divided into two sub-phases (Figure 15). The early storage phase S-I is covered by the advance phase model and the downstream boundary remains an advancing tip in the virtual field. At the real lower boundary of the furrow \( x = x_L \) the discharge \( Q_L(t) \) and the flow depth \( h_{w,L}(t) \) are calculated by Equations 73 and 69. Both \( Q_L \) and \( h_{w,L} \) increase with time and the virtual wave concept yields a realistic approximation of the flow conditions at the furrow outlet during the early storage phase (cf. Figure 19). The advance rates during S-I are usually well above zero.

Apart from the numerical problems which are associated with low advance rates, the determination of the advancing tip by the advance model is likewise time-consuming because of the iterative scheme for solving the flow equations. For this reason, the concept is changed to that of classical ZI modelling in sub-phase S-II: the downstream boundary condition during storage now becomes a stationary field boundary at the furrow outlet, namely, Manning's equation for uniform flow:

\[ Q_L(t) = K_n \sqrt{S_0 A_L} R(A_L)^{2/3} \]  

(78)

The sub-phase S-II starts at the time \( t = t_u \), when the difference between the inlet and the outlet flow depth is small:

\[ h_{w,L}(t)/h_{w,0}(t) \geq r_h \]  

(79)

where \( h_{w,L} \) at \( x = x_L \), \( h_{w,0} = h_{w} \) at \( x = x_0 \) and \( r_h = 0.80..0.99 \).
At this time, the reasonable assumption of a uniform flow depth along the furrow can be applied as:

$$\bar{h}_{wl} = \frac{(h_{wl,0} + h_{wl,L})}{2}$$  \hspace{1cm} (80)

The cross-sectional area $A_x$ at $x = x_L$ is now replaced by the averaged cross-sectional area $\bar{A}(\bar{h}_{wl}) = (\bar{h}_{wl} / p_1)^{1/p_2}$. Inserting $\bar{A}$ and $\bar{h}_{wl}$ in Equation 78 leads to the formula for the calculation of the outlet discharge:

$$Q_L = K_{st} \sqrt{S_0 \bar{A} R(\bar{A})^{2/3}}.$$  \hspace{1cm} (81)

At the beginning of S-II, the large gradients of the infiltration rate, which are characteristic for the beginning of the infiltration process, continuously decrease at each location along the furrow. The infiltration rate converges to a constant intake rate after a certain soil-specific opportunity time $t = \tau \geq t_u$. This has been observed at several field experiments, e.g. Wöhling [1999], and Esfandiari and Maheshwari [2001]. Once the infiltration rate at the furrow outlet $q(x_L, t)$ has become constant, the infiltration rate for the entire furrow

$$Q_{inf}(t) = \int_{x=0}^{x=x_L} q(x, t) \, dx$$  \hspace{1cm} (82)

is also constant.

By introducing the sub-phase S-II, the iterative calculation of $x_{up}$ is avoided and, thus, computation time becomes significantly reduced. The difference between the calculated infiltration volume of the advance model (S-I) and the new averaged flow-depth approach (S-I and S-II) is negligible, as is subsequently shown.

**Impact of S-II simplifications on cumulative infiltration**

In order to analyze the impact of the process adequate simplification of the flow description in the sub-phase S-II, it is necessary to skip a few subsections and to employ the irrigation model FAPS, which is developed in subsection 3.3.5. The model utilizes the 2D Richards equation (HYDRUS-2D) during all the hydraulic phases of the surface flow.

An irrigation of a 100 m long furrow on silty loam is simulated by FAPS. For this scenario, the flow depth is calculated at six equidistant cross sections utilizing (1) the advance model alone (sub-phase S-I), and (2) the new approach (S-I and S-II) by choosing $r_h = 0.8$. The flow depth of run (1) and (2) are shown in Figures 21a) and 21b) respectively. Figure 21c) shows the cross-sectional cumulative infiltration of the two runs which is calculated by:

$$I_{inf}(t, x) = \int_{\tau=0}^{\tau=x_L} q(x, t) \, d\tau$$  \hspace{1cm} (83)

Differences between corresponding $I_{inf}(t, x)$-values, although small, increase from the upstream sections to the downstream sections (Figure 21c). The integration of $I_{inf}(t, x)$ along the furrow length $x_L$ gives the cumulative infiltration of the entire furrow $V_{inf}(t, x)$. The corresponding $V_{inf}(t)$-values of the two runs are almost indistinguishable as seen in Figure 21d). Although these differences are well below 1.0%, the differences in $I_{inf}(t, x)$ may unfortunately falsify important performance criteria, for example, the calculated distribution uniformity.
The criterion $h_r = 0.8$, however, is quite weak. If $h_r = 0.9$ is chosen, the differences between $I_{inf}(t,x)$ of run (1) and (2) are also negligible at all cross sections.

The choice of the $h_r$-value is dependent on the desired model output; it is sufficient to choose $h_r = 0.8$ if only cumulative values of the total infiltration from the furrow are required. Otherwise, it is recommended to choose $h_r \geq 0.9$ in order to ensure a realistic prediction of the uniformity of water application along the furrow reach.

### 3.3.2.3 Depletion and recession phase model

A depletion and recession phase model is developed analogous to the recession model by Schmitz [1989] in close cooperation with Seus and Liedl. The model was first developed for the simulation of the recession phase and is subsequently extended for the depletion phase.

**Recession phase**

The flow velocity and flow rate in the furrow is small during the recession phase as compared to the flow velocity and flow rate during the advance and storage phase. Especially near the trailing edge of the surface water body, the vertical flow components (infiltration) may be
large compared to the horizontal flow components. By acknowledging these conditions, the surface flow domain is divided into two sections as seen in Figure 22.

Figure 22 Principles of the recession phase model

In **section (I)**, which is defined by $x_{\text{tail}}(t) < x < x_n(t)$, the flow is shallow and the flow velocity is usually very small. The water movement in the direction of the x-axis can be neglected and, thus, the surface water volume is only reduced by infiltration. In order to account for the ponding conditions, a horizontal water level is assumed.

In **section (II)**, which is defined by $x_n(t) < x < x_L(t)$, the flow in the direction of the x-axis is still dominant. Uniform flow conditions are assumed at the field outlet, just as during the late storage phase (S-II). The flow depth in section (II) is also uniform as seen in Figure 22 but time-dependent.

The common boundary of the section (I) and the section (II) at the intersection point $x_n$ is the common flow depth $h_n(t) = h_{nL}$ (cf. Figure 22).

The governing equations of the recession model are the continuity equation 55 and the simplified momentum equation:

$$\frac{\partial h_n}{\partial x} = S_0$$  \hspace{1cm} (84)

Further, the relations

$$\frac{\partial h_n}{\partial x} = 0 \quad \text{for} \quad x_{\text{tail}}(t) < x < x_n(t) \quad \text{and} \hspace{1cm} (85)$$

$$x_n(t) = x_{\text{tail}}(t) + \frac{h_n(A_n)}{S_0} \quad \text{for} \quad x_n(t) < x < x_L$$  \hspace{1cm} (86)

hold true. The wetting cross-sectional area $A_n$ at the point $x_n$, i.e. at the intersection of section (I) and (II), is derived by inserting $A_n$ in the solution of the Manning formula for uniform flow:
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\[ K_{irr} \sqrt{S_0} A_n R(A_n)^{2/3} = Q_L \]  

(87)

Initial and boundary conditions are

\[ A(x_{\text{tail}}(t), t) = 0 \]  

(88)

\[ Q(x_L, t) = Q_L \]  

(89)

\[ x_{\text{tail}}(t_r) = 0 \]  

(90)

where \( t_r \) denotes the start of the recession phase, \( Q_L \) the discharge at the location of the furrow outlet \((x_L)\). The equations to calculate cross-sectional area of the flow sections (I) and (II) are:

\[ A(x, t) = A_n \left( \frac{x - x_{\text{tail}}} {x_n - x_{\text{tail}}} \right)^{1/p_2} \quad \text{for} \quad x_{\text{tail}}(t) < x < x_n(t) \quad \text{and} \quad (91) \]

\[ A(x, t) = A_n \quad \text{for} \quad x_n(t) < x < x_L \]  

(92)

where

\[ A_n = \left[ \frac{S_n}{p_1} (x_n - x_{\text{tail}}) \right]^{1/p_2} \]  

(93)

The hydraulic radius at the location \( x_n \) is given by

\[ R(A_n)^{2/3} \frac{\partial h_{\text{wl}}}{\partial A} = c_{\text{surf}} A_{\text{surf}} \]  

(94)

and reconverted to

\[ R(A_n)^{2/3} = \frac{c_{\text{surf}}}{p_1 p_2} A_n^{(r_{\text{surf}} + 3 - p_2)} \]  

(95)

Equation 95 inserted into Equation 87 yields the flow rate at the furrow outlet:

\[ Q_L = K_{irr} \sqrt{\left( \frac{c_{\text{surf}} S_n}{p_1 p_2} \right) A_n^{(r_{\text{surf}} - p_2 + 3)}} \]  

(96)

The flow rate at the transition point \( Q_n \) is zero.

Depletion phase

The principles of flow description during the recession phase are also applied to the depletion phase by the following assumption.

At the beginning of depletion \( t = t_{\text{co}} \), the flow depth at the furrow inlet \( h_{\text{wl}}(x = 0, t = t_{\text{co}}) \) is assumed to extend horizontally in the direction of \(-x\) as seen in Figure 23. The position of the trailing edge is located at the intersection between the furrow bed and the horizontal water
Methods: the modules of GAIN-P

Figure 23   Depletion phase model

level $h_{inf}(x=0, t = t_{col})$. In an analogy to the advance model, the trailing edge of the surface water body $[x_{tail}(t_{co}) < (x = 0)]$ moves within a virtually extended field.

The Equations 84 – 96 also hold true for the depletion phase model if the initial condition Equation 90 is replaced by

$$x_{tail}(t_{co}) = \frac{h_0}{S_0} \tag{97}$$

To guarantee the continuity of both the depletion and the recession phase models, the volume balance reads:

$$V(t_{co}) - V_{surf}(t) = \int_{t_{col}}^{t} Q_L + V_{inf}(t) \tag{98}$$

where

$$V_{surf}(t) = \int_{x_{tail}(t)}^{x_c} A(t) \, dx \quad \text{and} \quad \tag{99}$$

$$V_{inf}(t) = \int_{t_{co}}^{t} \int_{x_{tail}(t)}^{x_c} q(\hat{x}, \tau) \, d\hat{x} \, d\tau \tag{100}$$

As a result, a closed-form description of the surface water flow during both the depletion phase and the recession phase is derived.

3.3.3   Subsurface flow model HYDRUS-2D

Physically based models for simulation of the subsurface water flow usually utilize some form of the Richards equation. The 2D Richards equation is also used by the numerical code HYDRUS-2D, which allows the simulation of both the infiltration from arbitrary shaped furrows and the soil moisture transport in a vertical plane. HYDRUS-2D is implemented in the new furrow irrigation model as described later in the text (subsection 3.3.5).
3.3.3.1 Governing equation

HYDRUS-2D is a powerful, widely used tool for the simulation of 2D water (and solute) transport problems [Simůnek et al., 1996]. The variably saturated water flow in a vertical plane is described by the modified 2D Richards equation [Vogel and Hopmans, 1992; Simůnek et al., 1996; Diersch and Perrochet, 1999]:

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial y} \left( K(h_u) \frac{\partial h_u}{\partial y} \right) + \frac{\partial}{\partial z} \left( K(h_u) \frac{\partial h_u}{\partial z} + K(h_u) \right) - s_r
\]

where

- \( y \) = the horizontal space coordinate perpendicular to the direction of the surface flow \( x \)
- \( z \) = the vertical space coordinate
- \( s_r \) = a sink term, which represents the volume of water removed per unit time from a unit soil volume due to plant water uptake.

3.3.3.2 Space discretization

The HYDRUS-2D flow domain is divided into a network of triangular elements. The corners of these elements are taken to be the nodal points of the calculation mesh. The mesh can map any furrow cross section by adapting the nodal locations according to the furrow geometry. In addition to the high flexibility of the mesh, the node density can be increased/decreased at locations of expected high/low pressure gradients, e.g. a high node density below the furrow and a low node density in deeper soil layers. This adoption of the calculation mesh can lead to greater numerical stability and more accurate simulation results. Essentially, the number of elements in the flow domain influence the model performance with respect to CPU-time consumption.

3.3.3.3 Initial and boundary conditions

The solution of Equation 101 requires the initial pressure head values at the nodes of the calculation mesh \( h(x,y,z) \) for \( t = 0 \). Three types of conditions are implemented in HYDRUS-2D in order to describe system-independent interactions along the boundaries of the flow region, namely, the pressure head type, the flux type and the gradient type boundary condition. Additionally, three different types of system-dependent boundary conditions are implemented. The atmospheric boundary, i.e. the soil-surface-air interface, is of special significance for irrigation modelling. The potential water flux across this interface (infiltration or evaporation) is controlled exclusively by external conditions. The atmospheric boundary is:

- a prescribed flux boundary condition in the case of precipitation and evaporation (directed either in or out of the flow domain), or
- a prescribed head boundary condition at times of irrigation and for \( h_{\text{surf}} \leq h_{\text{crit}} \).

Depending on the simulated pressure head at the atmospheric boundary nodes, the conditions may switch from flux to head and vice versa.

Another system-dependent boundary condition is applied at the lower boundary of the flow domain, namely, the seepage face type (for details cf. Simůnek et al. [1996]). Further, it is assumed that no lateral water flow occurs between adjacent furrows (no-flux condition). For details of the so-called zero-flux plane refer to Wöhling et al. [2004a].
3.3.3.4 Numerical solution techniques

To solve Equation 101, the Galerkin finite element method is used together with simplifying assumptions [cf. Simunek et al., 1996] in order to obtain a system of time-dependent ordinary differential equations. The integration of these equations in time is achieved by a discretization of the time domain into a sequence of finite intervals. An implicit finite difference scheme is used for both saturated and unsaturated conditions. The final set of nonlinear algebraic equations is solved iteratively for each time step. A detailed description of the solution technique can be found in the HYDRUS-2D handbook [Simunek et al., 1996].

3.3.3.5 HYDRUS-2D source code modification

The numerical code HYDRUS-2D is written in Fortran-77 language and designed for Windows-based operating systems. It was kindly made available for the present contribution by one of the authors of the program, Jirka Šimunek. In order to account for the special requirements which arise from the simulation of a complete growing season, some source code modifications are necessary. These modifications are conducted in the frame of the presented paper.

In order to fit HYDRUS-2D into the model development environment, the code language has been changed to Fortran-95 and assembled for the Matlab environment. The kernel (solver) of the code remains unchanged, but much of the parameter initialization, the variable assignment and the time management is now controlled by new external routines. Another essential modification is the implementation of an externally controlled (variable) atmospheric boundary.

The change of the boundary type of individual atmospheric boundary nodes during the simulation is not supported by the original HYDRUS-2D code. However, this change is necessary for simulating subsequent irrigation/redistribution times as well as the transient flow depth in a furrow cross section during irrigation. An interface routine has been developed, which assigns both boundary type and value to each of the atmospheric boundary nodes according to the transient boundary conditions, which are externally determined by the surface flow module and the evapotranspiration module. The interface routine is coupled to an event-control and time-management unit. This unit dictates that the interface routine is only executed at those times when the atmospheric boundary conditions change, rather than at every HYDRUS-2D time step. Section 3.3.7 gives an overview of the complex interaction of the control routines within the frame of the new furrow irrigation model.

3.3.3.6 Sensitivity and error analysis

A sensitivity and error analysis is subsequently conducted for HYDRUS-2D. The performance of the model is analyzed with respect to mass conservation, discretization errors and CPU-time requirements.

Various test runs are performed for 2D infiltration from a semi-circular furrow (with radius \( r = 0.175 \) m, flow depth \( h_{wl} = 0.15 \) m = const.) in a 2.5 x 2.5 m vertical soil plane of Lavalette silt loam (Table 19) which is initially assumed to be in equilibrium with an imposed matric head of \( h_m = -6.54 \) m at the bottom of the plane. For the various infiltration runs, the cumulative infiltration \( I_{inf} \) is used as comparative performance criteria in the analysis. The simulation time is four hours.
Discretization error and mass conservation
The discretization of numerical models usually has a strong impact on the model results. In order to analyze the impact of discretization on calculated infiltration and mass conservation, a large number of comparative test runs are performed (cf. Table 4) and subsequently analyzed.

Table 4 Sensitivity analysis of HYDRUS-2D corresponding time and space discretization for the test runs

<table>
<thead>
<tr>
<th>Model</th>
<th>Quantity</th>
<th>Discretization values</th>
</tr>
</thead>
<tbody>
<tr>
<td>HYDRUS-2D</td>
<td>Number of calculation nodes</td>
<td>[76, 148, 251, 515, 834, 1086, 1635] (nodes in the FE-mesh)</td>
</tr>
<tr>
<td></td>
<td>$\Delta t$ [hours]</td>
<td>Adaptive time step control or selected time steps of: [0.0001, 0.0002, 0.0005, 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1]</td>
</tr>
</tbody>
</table>

The calculated cumulative infiltration $I_{\text{inf}}(t)$ is more sensitive to space discretization (i.e. the number of the nodes in the calculation mesh) for time steps $\Delta t \leq 0.1$ hours. The coefficient of variation

$$
\nu_{I_{\text{inf}}} = \frac{\sigma}{\mu} \cdot 100\% \tag{102}
$$

of the cumulative infiltration at $t = 4.0$ hrs is $\nu_{I_{\text{inf}}}^{\text{HYDRUS-2D}} = 6.9\%$ for both the 70 test runs using 10 fixed time step lengths and for the 7 corresponding runs using an adaptive time step control (Table 4). The latter feature reduces both the computation time and the volume balance error $E_{V}^{\text{HYDRUS-2D}}$ significantly. For the test runs with adaptive time step control, the maximum of $E_{V}^{\text{HYDRUS-2D}}$ is determined to 0.6 %, whereas it is one order of magnitude higher for the simulations with the fixed time step length.

CPU time requirement
A comparison of CPU time and memory (RAM) requirements is conducted for a relatively coarse and dense HYDRUS-2D calculation mesh which features (1) nodal spacing of about 0.08 – 0.20 m (515 nodes) and (2) nodal spacing of about 0.01 – 0.10 m (1635 nodes), respectively.

For both examples (1) and (2), the required CPU-time of 100 separate runs is averaged. Furthermore, the mean CPU-time is standardized based on the mean CPU-time requirement by the coarse mesh parameterization:

$$
CPU \text{ – time factor} = \frac{CPU \text{ – time}}{CPU \text{ – time}_{515\text{nodes}}} \tag{103}
$$

The simulation with the fine mesh of 1635 nodes takes on average 7.7 times longer than the simulation with the coarse mesh of 515 nodes. However, the required memory increases only by the factor 1.6 (Table 5).
Table 5  CPU-time and RAM requirements of two HYDRUS-2D calculation meshes

<table>
<thead>
<tr>
<th></th>
<th>Space discretization</th>
<th>CPU-time factor</th>
<th>Memory factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example (1)</td>
<td>mesh with 515 nodes</td>
<td>1</td>
<td>1.0</td>
</tr>
<tr>
<td>Example (2)</td>
<td>mesh with 1635 nodes</td>
<td>7.7</td>
<td>1.6</td>
</tr>
</tbody>
</table>

3.3.4 Crop growth model

In the context of this study, 'crop growth modelling' comprises (i) the calculation of daily values of potential transpiration and evaporation, (ii) the prediction of the daily leaf-area index and (iii) the estimation of both the potential and the actual yield at the end of the growing period. These individual modules are described in the subsequent subsection and integrated in the crop growth model LAI-SIM (cf. also subsection 3.3.6).

3.3.4.1 Structure of evapotranspiration – potential crop transpiration

The evapotranspiration ET is the composite loss of water to the air from evaporation (EP) and transpiration (TP) [Oke, 1987]. The distribution of the two evapotranspiration components is called the 'structure of evaporation' [Budagovskij, 1969]. Daily values of the potential transpiration of a non-reference crop are calculated by multiplying the daily values of (site specific) reference evapotranspiration $ET_0$ with a splitting coefficient $C_o$ and the crop coefficient $K_c$ [Novak, 1981; Mailhol et al., 1997]:

$$TP(das) = C_p(das) \cdot K_c(das) \cdot ET_0(das)$$  \hspace{1cm} (104)

with the number of days after sowing $das$. The potential soil evaporation is calculated as:

$$EP(das) = (1 - C_p(das)) \cdot ET_0(das)$$  \hspace{1cm} (105)

with the splitting coefficient $C_p$ denoting the total area of leaves and other green parts of the plants related to a unit surface area:

$$C_p(das) = 1 - e^{-\alpha_w \cdot LAI(das)}$$  \hspace{1cm} (106)

with $das = [1, 2..nsd]$ and the total number of simulation days $nsd$. The specification of the plant coefficient $\alpha_w$ is ambiguous in literature. Kutilek and Nielsen [1994] state that $\alpha_w$ is probably not universally applicable throughout a growing season because daily values of TP and EP depend largely on the plants ontogenesis stage. Cambell [1985] suggested a general value of $\alpha_w = 0.82$, whereas other researchers found crop specific values: $\alpha_w = 0.7$ for corn [Varlet-Grancher et al., 1982], $\alpha_w = 0.75$ for cow beans, $\alpha_w = 0.6$ denotes an averaged value for wheat, cotton and sorghum [Sepaskhah and Ilampour, 1995]. Similar to the approach of Mailhol et al. [1997], an averaged value $\alpha_w = 0.7$ is used subsequently, which gives $EP/TP$ values concurring which those proposed by Ashktorab et al. [1994].

The crop coefficient $K_c$ can be interpreted as a scaling factor, denoting the difference between the transpiration characteristics of a crop and the reference crop respectively and is calculated as proposed by Allen et al. [1998] with modifications by Novak [1981] and Kutilek and Nielsen [1994]:
where $K_{c,max}$ is the maximum possible value of $K_c$ for a crop depending on local site conditions. $K_c$ can be smaller than unity for not fully developed crops and can reach up to 1.5 for fully developed vegetation. $K_c = 1$ is obtained when the crop transpiration rate equals the transpiration rate of the reference grass. Typical ranges of crop coefficients for some crops and their development stages are listed in Doorenbos and Kassam [1986].

### 3.3.4.2 Leaf-area index prediction

For field crops, the development of leaf area has a major influence on the production of biomass and yield [Chapman et al., 1993]. The structure of evapotranspiration depends also on the leaf-area index during the growing season. Daily leaf-area index values are calculated by the approach of Mailhol et al. [1997, 2004] assuming that all production factors are at an optimum and that only water stress reduces crop development:

$$\text{LAI}(\text{das}) = \text{LAI}_{\text{max}} \left[ \frac{TT(\text{das}) - T_b}{T_f} \right]^\beta \exp \left\{ \frac{\beta}{\delta} \left[ 1 - \left( \frac{TT(\text{das}) - T_b}{T_f} \right)^\delta \right] \right\} - \left(1 - (\text{stress}(\text{das}))^\delta \right)$$

with

$$TT(\text{das}) = \text{thermal time in degree-days [°C} \cdot \text{d]}$$

$$T_b = \text{the base temperature of the crop}$$

$$T(\text{das}) = \text{the daily mean air temperature}$$

$$\text{LAI}_{\text{max}} = \text{the maximum value of the leaf area index}$$

$$T_s = \text{the thermal time of emergence}$$

$$T_f = \text{the threshold thermal time corresponding to LAI}_{\text{max}}$$

$$\text{stress}(\text{das}) = \text{the water stress factor}$$

$$\lambda = \text{a parameter governing the plant sensitivity to water stress}$$

$$\beta, \delta = \text{two parameters related to the slope of the LAI curve.}$$

In order to account for a faster LAI decrease at times after senescence, the shape parameter $\delta$ is subdivided in $\delta_s$ and $\delta_2$ which account for $TT(\text{das}) < T_{\text{sen}}$ and $TT(\text{das}) \geq T_{\text{sen}}$, respectively. $T_{\text{sen}}$ denotes the thermal time of initial plant senescence and $T_{\text{sen}} \approx T_{\text{mat}} + 500$ [°C · d] can be assumed, where $T_{\text{mat}}$ denotes the thermal time at plant maturity.

The maximum leaf-area index $\text{LAI}_{\text{max}}$ is usually given for a specific optimum plant density $\text{densopt}$ [plants/m²]. It is scaled in LAI-SIM according to the input variable $\text{dens}$, denoting the actual plant density. Where yield is especially sensitive to crop-water stress, an interval is defined by $T_{\text{crit1}} \leq TT \leq T_{\text{crit2}}$, with $T_{\text{crit1}}$ and $T_{\text{crit2}}$ in [°C · d]. Table 6 shows a list of the specific parameters for some common crops as found by Mailhol et al. [1997], Mailhol [2003].
3.3.4.3 Crop yield

The estimation of the actual crop yield is of particular interest for the practitioner. From the theoretical point of view it is also a convenient integral measure for analyzing the effect of soil-water limitations during the simulation of the growing season.

A simple linear crop-water production function is recommended in the FAO Paper No. 33 [Doorenbos and Kassam, 1986] for predicting the reduction in crop yield when crop stress is caused by a shortage of water:

\[
1 - \frac{Y_a}{Y_m} = K_r \left( 1 - \frac{ET_{c,a}}{ET_c} \right)
\]

with

- \(Y_a\) = the actual crop yield
- \(Y_m\) = the maximum (expected) yield in absence of water and other environmental stress
- \(K_r\) = the yield response factor
- \(ET_c\) = the potential (expected) crop evapotranspiration in absence of water and other environmental stress
- \(ET_{c,a}\) = the actual (calculated) crop evapotranspiration as a result of environmental and/or water stress.

Replacing crop evapotranspiration with crop transpiration and rearranging Equation 110 leads to

\[
Y_a = Y_m \left[ 1 - K_r \left( 1 - \frac{\sum_{d=1}^{n_{das}} TA(das)}{\sum_{d=1}^{n_{das}} TP(das)} \right) \right]
\]
Irrigation control: towards a new solution of an old problem

$K_y$ values for selected crops are listed in Table 7. Others can be found in the FAO Paper No. 33 for both single crop growth stages and the total growing season.

Table 7 Yield module parameters

<table>
<thead>
<tr>
<th>Crop</th>
<th>Yield response factor $K_y$ [-]</th>
<th>Harvest Index $HI_{opt}$ [-]</th>
<th>Radiation use efficiency RUE [g· MJ$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maize</td>
<td>1.25</td>
<td>0.52</td>
<td>1.32</td>
</tr>
<tr>
<td>Sorghum</td>
<td>0.90</td>
<td>0.45</td>
<td>1.15</td>
</tr>
<tr>
<td>Potato</td>
<td>1.10</td>
<td>0.60</td>
<td>1.00</td>
</tr>
<tr>
<td>Soybean</td>
<td>0.85</td>
<td>0.31</td>
<td>0.48</td>
</tr>
<tr>
<td>Sunflower</td>
<td>0.80</td>
<td>0.37</td>
<td>0.68</td>
</tr>
<tr>
<td>Wheat</td>
<td>1.15</td>
<td>0.42</td>
<td>1.10</td>
</tr>
</tbody>
</table>

For describing yield reduction, the more adaptive approach of Mailhol et al. [1997] uses the ratio between potential and simulated LAI values during a period which is critical for the yield:

$$Y_a = Y_m \cdot \min\left(1, \frac{LAI_{av}}{LAI_{opt}}\right)$$  \hspace{1cm} (112)

where $LAI_{av}$ is the average actual LAI calculated during the critical period and $LAI_{opt}$ the average optimum value during the same period for obtaining the maximum yield. Critical periods for crop yield are identified for different crops in FAO Paper Nos. 24 and 33 [Doorenbos and Pruitt, 1992; Doorenbos and Kassam, 1986].

Both Equations 112 and 111 are implemented in LAI-SIM. Equation 112 is the more reliable (and thus preferred) approach, provided that experimental data for crop development under stressed and non-stressed conditions are available. If these data are not available, or if more than one specific time period during the growing season can seriously affect crop yield, Equation 111 can offer a reasonable estimation.

Potential yield

A rather simple approach to calculate the potential (expected) yield in absence of environmental and water stress $Y_m$ is used by Mailhol et al. [1997]:

$$Y_m = HI \cdot RUE \sum_{das=1}^{ndas} SR(das) \cdot ISR(das)$$  \hspace{1cm} (113)

with

$HI$ = the harvest index

$RUE$ = the radiation use efficiency

$SR$ = the daily incident solar radiation in [J m$^{-2}$]

$ISR$ = the fraction of solar radiation intercepted by the crop, which is estimated by

$$ISR(das) = 1 - e^{-c_{rad} \cdot LAI(das)}$$  \hspace{1cm} (114)

using an extinction coefficient
Methods: the modules of GAIN-P

\[ c_{ext} = \min \left[ 1.0, 1.43 \cdot (LAI(das))^{-0.5} \right] \]  

(115)

The harvest index \( HI \) denotes the harvested part of the crop, e.g. for cereal grains, relative to the total above-ground biomass production. It should be noted that \( HI \) in this context means the potential harvest index \( HI_{opt} \) which appears under optimum water (and nutrient) supply. \( HI_{opt} \) values of some crops are given in Table 7. Values of crops which are not included in the table may be found in Neitsch et al. [2002].

Mailhol et al. [2004] adapted their approach to calculate \( Y_m \) by introducing a functional relationship between the harvest index and the mean water stress during the critical period. It is assumed that \( HI \) decreases linearly from its potential value when the average LAI during this period \( LAI_{av} \) falls beneath a threshold value \( LAI_{st} \):

\[ HI_{red} = \min \left[ HI_{opt}, \left( HI_{opt} - a, \left( LAI_{st} - LAI_{av} \right) \right) \right] \]  

(116)

Apart from \( LAI_{st} \), the second empirical parameter for this adapted approach is the decrease factor \( a \). Although this approach is recommended (and applied) for crops with well-documented field experiments, it is not used in the context of this paper because the two additional empirical parameters are not generally accessible for all considered crops and model applications.

The radiation use efficiency \( RUE \) quantifies the efficiency of a plant for converting light energy into biomass. The determination of \( RUE \) is performed by standard methods (e.g. Kiniry et al. [1999]) and values for different crops are found in technical literature (e.g. Neitsch et al. [2002]). \( RUE \) is not constant during the growing period and can be associated with a change in LAI or temperature or other environment factors such as nitrogen (N) and soil-water content (e.g. Neitsch et al. [2002]). An approximate mechanistic simulation of the change in \( RUE \) after anthesis and the absence of N and temperature limitations, however, can be compensated for by the use of an averaged \( RUE \) value [Mailhol et al., 1997] in the context of this study (Table 7).

3.3.5 Coupling surface and subsurface flow

Surface flow and subsurface flow are closely interlinked in irrigation modelling. Both surface and subsurface flow are highly dynamic processes and the mathematical complexity increases immensely by coupling them. Surface and subsurface flow in the furrow irrigation models by Strelkoff and Katopodes [1977], Elliott and Walker [1982], Ross [1986], SIRMOD III [2003] interact solely by the infiltration rate. However, the flow depth also plays an important role in transporting and distributing water (and solutes) in the soil profile [Abbasi et al., 2003]. On the one hand, the flow-depth impact on infiltration can be considered by using a 2D infiltration model. On the other hand, the solution of the resulting set of non-linear flow equations becomes much more elaborate numerically as compared to the use of the Kostiakov equation only.

In this section, the analytical ZI surface model is coupled with both HYDRUS-2D and, in order to compare simulation results with those of other models, the Kostiakov equation. The mathematical coupling method used is the alternating iterative coupling according to the classification by Morita and Yen [2002]. Acknowledging the need for a numerically stable
and convergent solution for the set of partial differential equations, a new solution method is presented and compared with the common solution of the considered problem.

### 3.3.5.1 Principles of alternating iterative coupling

Alternating iterative coupling is still a challenging task. Often, the modeller is confronted with numerical instability and poor convergence behaviour, especially during the simulation of irrigation advance. Morita and Yen [2002] define alternating iterative coupling as follows. In the coupled model, surface and subsurface equations are solved separately but iteratively at the same time step, interlinked by the infiltration rate as the common internal boundary condition between surface and subsurface flows. When the iterations for both surface and subsurface flow converge within specific tolerances at a certain time step, the computation advances to the next time step. These principles are largely followed in this study.

Figure 24 shows the mutual interaction of variables between surface and subsurface flow equations of the presented flow models for a certain time step of the advance phase. The infiltration rate is also a term of the surface flow Equation 55 and calculated by the subsurface flow model. At predefined locations along the furrow, the cross-sectional infiltration rate $q = f(\tau(x,t))$ is calculated by either the Richards or Kostiakov equation and subsequently integrated over the wetted furrow distance $[x_0,x_{tip}(t)]$, which yields the total infiltration rate from the surface water body $Q_{inf}(t)$. In contrast to $Q_{inf}(t)$, which only depends on time, the cross-sectional infiltration rate is dependent on infiltration opportunity time $\tau(x,t)$, which, in turn, is both time and space-dependent. During irrigation advance, the infiltration opportunity time at the field inlet is equal to the simulation time $\tau(x_0) = t$ and decreases (nonlinear) downstream towards the wave tip, where $\tau = 0$. In contrast to the infiltration rate of the empirical Kostiakov equation $q_k$, the infiltration rate calculated by the Richards equation $q_R$ not only depends on the wetted furrow reach and infiltration opportunity time. It is likewise dependent on the flow depth $h_{wlh}(x,t)$ because the total flux from the cross section is calculated by integrating nodal fluxes at the wetted furrow perimeter $wp(x,t)$. In mathematical terms, this additional dependency makes a successful coupling of surface flow and 2D infiltration much more difficult. As reported by Schwankl and Wallender [1988], the determination of the influence of flow depth on infiltration volume is a critical issue in surface irrigation modelling. With $A(x,t) = f(A_0,t)$ (Equation 65) and $h_{wlh}(x,t) = f(A(x,t))$, the infiltration rate $q_R(x,A_0,t)$ finally depends on the cross-sectional area $A_0(t)$ at the field inlet (Figure 24). The value of $A_0(t)$ together with the advance trajectory $x_{tip}$, however, is subject to the analytical solution of the surface flow equations Equation 65.

These mutual dependencies between surface and subsurface flow equations add much to the mathematical complexity of the coupling problem and can cause the numerical instability of the solution. Most mathematical problems have to be reckoned with when modelling the irrigation advance phase because:

- the irrigated field is only partly covered with water during advance. The location of the wetting front is both the downstream boundary of the surface flow and part of the solution of the flow equations.

---

8 Under some circumstances the receding water front (recession phase) can compensate some of the differences in opportunity time between inlet and outlet location.
irrigation on dry soil is associated with steep gradients in flow depth. Also, the hydraulic gradient between surface water and initially dry soil is usually very large. This can cause numerical instabilities for solving the Richards equation. Furthermore, the gradients of the infiltration rate are usually large during the advance phase and small during later phases of an irrigation event.

For the above reasons, the following subsections place particular emphasis on the description of the coupled surface and subsurface flow equations during irrigation advance. Uniform surface flow conditions are assumed during the storage phase S-II, the depletion and the recession phase (subsections 3.3.2.2/3.3.2.3). During these phases the surface flow and the subsurface flow are consistent as regards mass continuity.

### 3.3.5.2 Coupling FAP advance model and infiltration

The analytical zero-inertia surface irrigation model FAP (subsection 3.3.2.1) and the numerical code HYDRUS-2D (subsection 3.3.3) are coupled in the Matlab environment. In order to compare with other furrow irrigation models, the simple Kostiakov equation is also employed in FAP. For convenience, the model versions utilizing HYDRUS-2D and the Kostiakov equation are distinguished by the extensions -H and -K respectively, e.g. FAP-H and FAP-K.

In this subsection, the calculation of infiltration from the wetted furrow reach, the principle aspects of the fixpoint iteration scheme and the individual steps of the developed iterative coupling are described. Finally, the numerical behaviour of the coupling scheme is investigated in an example irrigation.

**Iterative calculation of infiltration from the wetted furrow reach**

HYDRUS-2D is employed at different user-defined (equally spaced) infiltration sections \( x_{inf} \) along the furrow (Figure 20). This leads to a series of infiltration models, which are characterized by their individual boundary and initial conditions and the local soil properties.
A visualization of the surface flow channel together with the finite element (FE) mesh grids of the employed 2D subsurface flow models is shown in Figure 25. The actual field dimensions (furrow length) are confined by the outer infiltration sections. The left furrow extension of Figure 25 indicates the virtual field as described in subsection 3.3.2.1. Five infiltration sections are distributed with equidistant space increments along the real furrow length, thus dividing it into quarters. This discretization is preferable and necessary for the calculation of the irrigation performance indices \( DU_{1q} \) and \( AD_{1q} \). 9

The first time step of the surface flow simulation requires an initial estimate of the location of the advancing wave front. This is estimated without computing the infiltration. The flow depth resulting from this initial estimate of \( x_{\text{tip}} \) initializes the iterative procedure (Figure 26) by providing the boundary conditions of the 2D infiltration computations which determine the cross-sectional cumulative infiltration \( I_{\text{inf}}(t, x) \). Similarly, for all subsequent time steps in the calculation, FAP provides the boundary condition for HYDRUS-2D by calculating the flow depth along the furrow.

The total volume of infiltrated water \( V_{\text{inf}}(t) \) is computed by linear interpolation along the wetted furrow reach:

\[
V_{\text{inf}}(t) = \sum_{i=1}^{\text{index}} \left( \Delta x_{\text{inf}} \cdot I_{\text{inf}}(t, x_{\text{inf}}(i)) \right) + \frac{I_{\text{inf}}(t, x_{\text{inf}}(\text{index}))}{2} \left( x_{\text{tip}}(i) - \text{index} \cdot \Delta x_{\text{inf}} \right)
\]

with

- \( I_{\text{inf}}(t, x_{\text{inf}}(\text{index})) \) = the cumulative infiltration at the last flooded infiltration section downstream of the furrow, and
- \( \Delta x_{\text{inf}} \) = the distance between successive cross sections \( x_{\text{inf}} \) where 2D infiltration is calculated.

---

9 These performance criteria can also be calculated for a multitude of quarters, i.e. if the number of infiltration sections is \([9,17,...]\), provided that they are distributed by equidistant space increments.
Methods: the modules of GAIN-P

The impact of the longitudinal resolution of the infiltration computations on $V_{inf}$ is described in detail by Wöhling et al. [2004b]. Since $V_{inf}$ is a variable of both the surface and subsurface flow equations, it plays an important role in the numerical coupling scheme as is subsequently shown.

**Coupling via fixpoint iteration**

The abstract form $\begin{bmatrix} A_0 \\ x_{tip} \end{bmatrix} = \mathbf{F}(x_{tip}, A_0)$ of the Equations 76 and 77 suggest the use of a fixpoint iteration to determine $A_0^{(k)}$ and $x_{tip}^{(k)}$. The corresponding scheme is relatively simple because derivatives of the functions are not involved. According to the Banach Fixpoint Theorem (cf. Hanke-Bourgeois [2002]), the method converges linearly if the mapping $\mathbf{F}$ is contractive. In FAP, this scheme is applied with a suggested discretization of equidistant time intervals $\Delta t$ because the variables in the fixpoint equations are time-dependent.

Figure 26 Principles for coupling surface and subsurface models
Iterative surface-subsurface coupling scheme

The furrow irrigation model for irrigation advance is formed by Equations 76, 77 and 117, together with the iteration methodology described subsequently. In general, analytical models do not require a time discretization. However, the analytical surface flow model FAP is evaluated at selected time intervals $\Delta t$ in order to correctly calculate the transient quasi-3D infiltration from the irrigation furrow. For convenience, the time interval is equal to the discrete time step of the HYDRUS-2D calculations. It can be chosen with respect to the desired accuracy of the model results [Wöhling et al., 2004b] and is usually in the range of $\Delta t = [10\text{s}.. \ 120\text{s}]$.

The set of nonlinear equations 76, 77 and 117 is solved iteratively for the time levels $t_i = j \cdot \Delta t$ ($j = [1, 2, 3, \ldots nt]$; $nt =$ number of time steps). Four steps are distinguished in the iteration scheme which are portrayed in Figure 26 and described subsequently.

**Step a)** Initializing the variables: In the first iteration level (iteration count $k = 1$), the variables $V_{in}^{(k)}(t)$, $A_0^{(k)}(t)$, $q^{(k)}(t)$, $x_{tip}^{(k)}(t)$ are initialized according to the initial conditions. For all other iteration levels, the variables are updated using the results of the preceding iteration.

**Step b)** Infiltration: The cross sections which are actually flooded are determined using an index, which denotes the number ($i$) of the last flooded infiltration section before $x_{tip}^{(k)}(t)$ (Figure 20). The flow depth $h_w^{(k)}(t,x_{tip}(i))$ at the cross sections $i = [1, 2, \ldots \text{index}]$ provides the upper boundary condition of the HYDRUS-2D models; the lower condition is taken as a seepage face boundary type. The cross-sectional cumulative 2D infiltration $I_{w}^{(k)}(t,x_{inf}(i))$ is calculated by HYDRUS-2D and integrated along the furrow by Equation 117 in order to obtain the total infiltration volume $V_{inf}^{(k)}(t)$.

**Step c)** Evaluation of $A_0$ and $x_{tip}$: By inserting $V_{inf}^{(k)}(t)$ and $A_0^{(k)}(t)$ into the analytical solution Equation 76, the new value $A_0^{(k+1)}(t)$ is computed. This new value is inserted into Equation 77 for determination of $x_{tip}^{(k+1)}(t)$.

A relaxation is utilized, as is common for the solution of linear equations, with selected initial relaxation parameters $rel_A = rel_x = 0.8$ in order to avoid overshooting and the alternating of subsequent iterations in the fixpoint scheme:

$$
A_0^{(k+1)}(t) = rel_A \cdot A_0^{(k+1)}(t) - (1- rel_A) \cdot A_0(t-\Delta t)
$$

$$
x_{tip}^{(k+1)}(t) = rel_x \cdot x_{tip}^{(k+1)}(t) - (1- rel_x) \cdot x_{tip}(t-\Delta t)
$$

(118)

**Step d)** Convergence criterion: The relative difference of subsequent iterations of $x_{tip}(t)$ must be less than $\varepsilon$:

$$
\left| \frac{x_{tip}^{(k+1)}(t) - x_{tip}^{(k)}(t)}{x_{tip}^{(k+1)}(t) + x_{tip}^{(k)}(t)} \right| < \varepsilon
$$

(119)

where $\varepsilon$ denotes the precision criteria, set to $10^{-3}$.

If the scheme does not converge within the limit of $it_{max} = 20$ iteration steps, the relaxation parameter $rel_x > rel_{min}$ is adaptively reduced to improve the convergence behaviour of the fixpoint scheme. In this case, the iteration is continued with the values
of $A^{(k=l_{\text{max}})}_i(t)$ and $A^{(k=l_{\text{max}})}_{ij}(t)$ for another $l_{\text{max}}$ iteration. One must be aware of the fact that small relaxation parameters enforce convergence to the preceding iterate instead of the solution being looked for.

Once the calculated irrigation advance between subsequent iteration steps no longer changes significantly (Equation 119), convergence is achieved and the simulation progresses to the next time interval $\Delta t$.

It is unlikely that the flooding of the computational cross sections begins exactly at a new time interval. Because of this, the infiltration opportunity times $\tau_i$ at just flooded cross sections $x_{\text{inf}}(i)$ are approximated by linear interpolation, assuming a linear advance trajectory during the time interval:

$$
\tau_i = \frac{(x_{\text{tip}}(t_j) - i \cdot \Delta x_{\text{inf}}) \cdot \Delta t}{(x_{\text{tip}}(t_j) - x_{\text{tip}}(t_{j+1}))}
$$

where $\tau_i$ is the infiltration opportunity time at the already flooded $i^{th}$ cross section; $x_{\text{tip}}(t_j)$ and $x_{\text{tip}}(t_{j+1})$ denote the position of the wave front at the new and at the previous time level, respectively. Both FAP and HYDRUS-2D use the same time interval/step with the only exception of initially flooded cross sections $x_{\text{inf}}(i)$ with $\tau_i < t_j$, where the infiltration is computed with a time step of $\tau_i < \Delta t$.

The described fixpoint iteration scheme, together with the governing Equation 76, 77 and 117, lead to the quasi-3D water flow model for simulation of the irrigation advance in furrows.

**Numerical behaviour**

The set of nonlinear Equations 76, 77 and 117 in FAP-H is solved numerically. An appropriate numerical method needs to be convergent, consistent (i.e. continued iteration allows for a better approximation) and stable (i.e. small changes in the input values result in similarly small changes of the output). Using the fixpoint iteration method, only first-order convergence can be expected. Generally, poor numerical behaviour is either the result of applying an inappropriate solution method or of an inappropriate modelling of the processes involved.

Various test runs are conducted by FAP-H in order to analyze the numerical behaviour of the coupled model. The model performance is good for small and moderate infiltration intake of the soil [Wöhling et al., 2004b]. But if about 2/3 or more of the prescribed amount of irrigation water is infiltrating during irrigation advance (which, in fact, is often the goal in irrigation practice), the convergence behaviour of the fixpoint scheme is poor. In addition to the fixpoint iteration, alternative methods are tested for solving the nonlinear set of equations. For this reason, the Equations 76 and 77 are reformulated to a least-square problem, which is then solved by mathematical standard routines from the *Matlab Optimization Toolbox*. These are:

- `fminsearch/fminunc`: for finding the minimum of an unconstrained multivariable function
- `fmincon`: for finding the minimum of a constrained multivariable function.
None of the above solver routines, however, shows a better convergence behaviour for the parameter range under consideration. In addition, the computation time increases enormously, since the number of function evaluations is generally about ten times higher.\textsuperscript{10}

We also had no luck with our attempts to significantly improve the fixpoint iteration scheme by a 'better' choice of initial values for $x_{t+\Delta t}$ and $A_0(t+\Delta t)$. Likewise, a modified relaxation scheme, where relaxation parameter $rel \in [0.5 \leq rel \leq 1.0]$ depends on the difference between subsequent $x_{t+\Delta t}$-iterations, also fails to converge.

As a result of the above investigations, it seems almost certain that the choice of methodology for solving the set of equations is not the reason for poor convergence under the given circumstances.

Example problem

Example Cou-A: A 130 m long field of silty loam is irrigated with a constant inflow rate of 0.0005 m/s. The parameterization of the coupled surface model is: slope $0.0025 \text{ m/m}$, $p_1 = 1.1969$, $p_2 = 0.6303$, $p_3 = 0.4632$, $p_4 = 0.5579$, $K_{sat} = 25 m^{1/3}/s^{-1}$. The soil in five vertical infiltration sections of $0.8 \times 5.0 m$ is homogeneous with an initial matric head of $h_m = -30 m$. The soil hydraulic parameter of the VGM model are: $\theta_s = 0.38$, $\theta_m = 0.05$, $\alpha = 1.5 m^{-1}$, $n = 1.46$, $m = 1 - 1/n$ and saturated hydraulic conductivity is $K_s = 3.9 \times 10^{-6} m/s$. The infiltration sections $x_{inf}$ are located at $x = 0$, 32.5, 65.0, 97.5 and 130.0 m. The time step is chosen to $\Delta t = 30 s$. This scenario is subsequently referred to as Run 1.

Since the non-convergence problems are obviously not a question of solution methodology, the phenomena associated with these problems are analyzed by a 'close-up' of subsequent iterations during a test run with the following parameterization:

Figure 27a) shows the number of subsequent iterations $k_{cum}$ of Run 1 over the respective calculated advance tip $x_{cum}(t)$. After about 130 iterations, the infiltration section at $x_{inf}(i) = 32.5 m$ is flooded. Now $x_{cum}(t)$-values continue to oscillate for more than another 170 iterations, before the simulation proceeds to the next time step (Figure 27a). This oscillation, however, follows a systematic pattern, which is also observed for other test runs:

- $x_{cum}(i)$ is calculated to be close to, but smaller than $x_{inf}(i)$, i.e. $x_{inf}(i-1) < x_{cum}(i) < x_{inf}(i)$. The infiltration rate $q(x_{cum}, i)$ is still zero.

- At the subsequent iteration $x_{cum}(i+1)$ is calculated to $x_{cum}(i+1) < x_{inf}(i)$. Thus, the infiltration rate at $x_{inf}(i)$ is (very) high because infiltration opportunity time $\tau$ at $x_{inf}(i)$ is small.

- The resulting sudden increase in cumulative infiltration $I_{inf}(x_{inf}(i), t)$ leads to an increase of $V_{inf}(i)$, a term in the volume balance Equation 72. Since the volume balance has to be satisfied at all times, the estimate for $x_{cum}(i+2)$ is decreased and consequently falls back behind $x_{inf}(i)$ so that $x_{cum}(i+2) < x_{inf}(i)$. Now the loop starts from the beginning again.

\textsuperscript{10} In terms of computational effort, the number of function evaluations can be considered equal to the number of iterations in the fixpoint iteration scheme.
A close look at the function of the cumulative infiltration $V_{\text{int}}(t)$ reveals its discontinuous character at the times of flooding of the individual cross sections (in fact, $Q_{\text{int}}(t)$ is discontinuous). As seen in Figure 27a) the refinement of the resolution of infiltration calculations along the furrow by $\Delta x_{\text{inf}} = 5.0$ m (Run 2) also fails to solve the problem; upcoming oscillations are only shifted in time. In the first case (Run 1), the simulation proceeds to the next time step only because the relaxation parameter $rel_x$ falls below the convergence criteria $\varepsilon = 10^{-3}$ (as a consequence of the non-convergence rule described in step d) of the coupling scheme). Relaxation parameters below a certain limit $rel_{\text{min}}$, however, enforce convergence to the preceding iterate and, thus, result in a linearly increasing advance rate as seen in Figure 27b). Of course, these simulation results are highly erroneous and do not at all represent the physical flow processes.

Attempts at simulations without relaxation or with invariant relaxation parameters are not successful either. In test Run 3, $rel_x$ is fixed at 0.7 and the simulation is forced to proceed after a maximum of 20 iterations to the next time step. As seen in Figure 27a), calculated $x_{\text{tip}}$ - values already start oscillating after three time steps. Small advance rates together with strong oscillations of $x_{\text{tip}}(t)$ can lead to calculated advance values which are smaller than the value at the previous time $x_{\text{tip}}(t - \Delta t)$ and, consequently, to aborted model termination, as is the case for Run 3 (Figure 27b).
In conclusion, the poor convergence behaviour of the fixpoint iteration for large inflow/infiltration fractions is mainly due to the superposition of numerical effects (discretization of $x_{\text{inf}}$ with mathematical assumptions linked to the surface flow process). The discontinuous $\frac{V_{\text{inf}}(t)}{Q_{\text{inf}}(t)}$-functions play a key-role, especially for small irrigation advance rates. It can be shown for these cases that the fixpoint method applied to Equations 76, 77 and 117 is not contractive and, hence, not convergent (Run 3 in Figure 27a). Interestingly, the convergence behaviour is also poor for corresponding test runs, where HYDRUS-2D infiltration is replaced by the simple Kostiakov equation. From the above findings, two preconditions for a more efficient/realistic solution and better convergence can be formulated:

- **A continuous $V_{\text{inf}}(t)$-function**
  
  The infiltration rate function $\int_0^{x_{\text{ip}}} q(\hat{x}, t) d\hat{x}(t, x_{\text{ip}} > x_{\text{inf}}(i))$ displays a discontinuity at those times when a cross section $x_{\text{inf}}(i)$ is flooded. This is a consequence of the high infiltration rate at a cross section with small infiltration opportunity times. Adequate refinement of $\Delta x_{\text{inf}}$ does not overcome the discontinuity but it does significantly increase the computational effort. This discontinuity is also transmitted to the cumulative infiltration $V_{\text{inf}}(t, x_{\text{ip}})$. For the preceding infiltration sections $x_{\text{inf}}(i-1) \leq x_{\text{ip}} \leq x_{\text{inf}}(i)$, however, $V_{\text{inf}}(t, x_{\text{ip}})$ is a continuous function. Consequently, the new solution algorithm must take this particular behaviour of the infiltration function into account.

- **Second-order convergence**
  
  The fixpoint iteration scheme is at most first-order convergent. The new solution algorithm should employ a second-order convergent scheme for solving the governing equations (more efficiently), like the Newton iteration method.

In view of these requirements, a new solution methodology for the set of governing equations is subsequently developed.

### 3.3.5.3 FAPS – from time to space discretization

It is realized that $x$ and $t$ can be considered as independent variables in the surface flow equations, i.e. instead of $A(t), x(t)$ the terms $t(x), A(t(x)) = A(x)$ can be evaluated. Here, in contrast to FAP methodology, a discretization of arbitrary space intervals along the furrow is chosen, which corresponds to the locations of infiltration computations $x_{\text{inf}}(i)$. For this type of space discretization, the set of governing equations 65, 71 is reformulated to calculate advance times rather than advance locations. This is similar to the concepts of Schwankl and Wallender [1988] and Schmitz et al. [2002]. The infiltrating water volume is now calculated at predetermined locations $x_{\text{inf}}(i) = x_{\text{ip}}(i)$ for varying opportunity times $t(x_{\text{ip}}(k))$ which correspond to the continuity intervals of $V_{\text{inf}}(t, x_{\text{ip}})$. This is essential for the application of gradient-based solution methods to the system of nonlinear equations like the Newton method.

Another advantage of the new approach is a major decrease in simulation time. Introducing the new space discretization, the number of iteration loops during the advance phase becomes equal to the number of infiltration sections $\text{index} - 1$. In contrast, the number of iteration loops for the predetermined time discretization in FAP is equal to the total advance time divided by the time step length. Consequently, the computational effort increases (rapidly) with decreasing advance rates for a given time step length.
The subsequently derived set of nonlinear equations is solved iteratively by means of the Newton method, which is one of the most common and effective schemes for solving systems of nonlinear equations. Newton's method is known to be quadratically convergent to the solution of the problem, assuming that the initial guess is sufficiently close to the solution and that the Jacobian Matrix of the system is not singular. Both assumptions are satisfied by the system under consideration. Since Newton's method is based on gradient information, (at least approximate) first order partial derivatives of the system of equations must be computed. This leads to a considerable increase in model complexity. Derivative approximations must be carefully chosen in order to guarantee the consistency of the model, as described later on.

In comparison, the simple fixpoint iteration which is applied to solve the time-discretized FAP model is, at most, first-order convergent, but does not require gradient information. The convergence strongly depends on the choice and accuracy of the initial guess. The contractivity of the system of governing equations is necessary for convergence, but cannot generally be improved upon by relaxation. Furthermore, the choice of the relaxation parameter is a tricky task, since small relaxation parameters considerably increase the computational effort and may even lead to wrong solutions.

The advantages and disadvantages of the new solution strategy, which is referred to as FAPS, compared to the time-discretization plus fixpoint formulation FAP, are subsumed in Table 8.

Table 8 Advantages and disadvantages of FAPS and FAP model features

<table>
<thead>
<tr>
<th>FAPS</th>
<th>FAP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Space discretization</strong></td>
<td><strong>Time discretization</strong></td>
</tr>
<tr>
<td>(+++) Yields a continuous $V_{inf}(t)$-function,</td>
<td>(--) Compatibility problems of surface/subsurface flow equations; $V_{inf}(t)$ implicitly/discontinuously depending on $x_{tip}$</td>
</tr>
<tr>
<td>(+) Computation time economical</td>
<td>(-) Computation time costly</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Newton Iteration Method</strong></th>
<th><strong>Fixpoint Iteration</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>(+) Second-order convergence</td>
<td>(+++) No derivatives of the gov. equations required</td>
</tr>
<tr>
<td>(+) No dependency on initial values observed</td>
<td>(o) Maximal first-order convergence</td>
</tr>
<tr>
<td>(-) Derivatives of the gov. equations required ⇒ more complex mathematical model</td>
<td>(-) Convergence strongly dependent on initial values</td>
</tr>
<tr>
<td></td>
<td>(-) Relaxation indicated</td>
</tr>
</tbody>
</table>

In general, the FAPS model is much more promising for both numerical efficiency and a good representation of the flow processes involved.

Reformulating governing equations
Advance time $t(x)$ and values of $A_0(t(x))$ are now required to calculate the wetted cross-sectional area of the furrow $A(x, t)$ and discharge $Q(x, t)$ for locations $x$ from Equations 65 and 71. These values are again determined by an iterative scheme, which, in contrast to the FAP iteration scheme, it is now based on a given discretization of the $x$-axis (along the furrow length): $0 < x_1 < x_2 < ... < x_N$
This discretization coincides with the discretization for the infiltration computations. The advance time $t_m$ is calculated for a given location $x_m$, i.e. $x_{tip}(t_m) = x_m$ and wetted cross section at the furrow inlet $A_0(t_m) = A_{0,m}$. The following equation for $A_{0,m}$ is obtained by using Equation 64:

$$A_{0,m} = \left(\frac{x_m}{c_s \alpha_s} \left[\frac{\bar{v}^2(t_m)}{K_m^2} - (S_0 + \frac{\bar{q}(t_m) \cdot \bar{v}(t_m)}{g \cdot A_m})\right]\right)^{\alpha_s}. \quad (121)$$

and corresponds to Equation (12) in Schmitz and Seus [1992]. The parameters $\bar{Q}_m$ and $\bar{R}_m$ are implicitly dependent on $A_{0,m}$ as follows:

$$\bar{A}_m = A_{0,m} \left(\frac{\alpha_s + 1}{\alpha_s + 2}\right)^{\alpha_s} \quad (122)$$

$$\bar{R}_m = p_A \bar{A}_m = p_A A_{0,m} \left(\frac{\alpha_s + 1}{\alpha_s + 2}\right)^{\rho \alpha_s} \quad (123)$$

$$v_m(t_m) = \frac{v_{tip}(t_m)}{(\alpha_s + 2)} - \left(\frac{\alpha_s + 1}{\alpha_s + 2}\right) \left(\int_0^{x_m} q(\hat{x},t_m) d\hat{x} - v_0(A_{0,m})\right) + \left(\frac{\alpha_s + 2}{\alpha_s + 1}\right) \left(\int_{x_m}^{x_p} q(\hat{x},t_m) d\hat{x}\right) \quad (124)$$

with

$$v_{tip}(t_m) = (\alpha_s + 1) \left(\frac{Q_0(t_m) - \int_0^{x_p} q(\hat{x},t_m) d\hat{x}}{A_{0,m}} - V_m(t_m) - V_{tip}(t_m) \frac{\partial A_{0,m}}{\partial t_m}\right) \quad (125)$$

$$v_0(t_m) = \frac{Q_0(t_m)}{A_{0,m}} \quad (126)$$

The Equation 122 for the calculation of the cross-sectional area at the centre of gravity $\bar{A}_m$ is obtained by Equation 65. The second necessary relation to determine $A_{0,m}$ and $t_m$ is provided by Equation 72.

Newton iteration scheme

One of the most common methods for the solution of (systems of) nonlinear equations of the form

$$F(x_1, \ldots, x_m) = \begin{pmatrix} f_1(x_1, \ldots, x_m) \\ \vdots \\ f_m(x_1, \ldots, x_m) \end{pmatrix} = 0 \quad (127)$$
Methods: the modules of GAIN-P

is the Newton method. This iterative method is known to be quadratically convergent to the solution of Equation 127 if the initial guess is sufficiently close to the solution. The convergence domain depends on the slope of $F$. To apply the Newton method, (at least approximate) first-order partial derivatives of $F$ must be computed. This can be a disadvantage of the method if approximate derivatives are not available or too expensive and time-consuming to compute. In order to apply Newton's method to the present problem, Equations 121, 122 and 72 are transformed to the following two-dimensional vector function with the independent variables $A_{0,m}$ and $t_m$:

$$F(\mathbf{A}_{0,m}, t_m) = \begin{pmatrix} f_1(\mathbf{A}_{0,m}, t_m) \\ f_2(\mathbf{A}_{0,m}, t_m) \end{pmatrix}$$

where

$$f_1(\mathbf{A}_{0,m}, t_m) = \frac{\bar{v}(t_m)}{K_{st}} - C_1 A_{0,m} - C_2 A_{0,m}^{\kappa} - C_3 \bar{v}(t_m) A_{0,m}^{\kappa-1}$$

and

$$f_2(\mathbf{A}_{0,m}, t_m) = \int_0^t Q_0(\tilde{t}) \, d\tilde{t} - \frac{x_m}{\alpha + 1} A_{0,m} - V_{inf}(t_m)$$

with the abbreviations

$$\kappa_s = \frac{4}{3} \rho_s,$$

$$C_1 = \frac{c_s \alpha_s}{x_m},$$

$$\mu_s = \frac{1}{\alpha_s},$$

$$C_2 = S_0 \rho_s^\prime \left( \frac{\alpha_s + 1}{\alpha_s + 2} \right)^{\kappa_s \alpha_s}$$

and

$$C_3 = \frac{\bar{v}(t_m)}{g} \rho_s^\prime \left( \frac{\alpha_s + 1}{\alpha_s + 2} \right)^{(\kappa_s - 1) \alpha_s}.$$

For each $x_m$, the roots $A_{0,m}, t_m$ of the equation $F(\mathbf{A}_{0,m}, t_m) = 0$ have to be determined. Starting with initial values of $A_{0,m}^{(0)}, t_m^{(0)}$, the following iteration scheme is applied:

$$\begin{cases} A_{0,m}^{(k+1)} \\ t_m^{(k+1)} \end{cases} = \begin{cases} A_{0,m}^{(k)} \\ t_m^{(k)} \end{cases} - \mathbf{J}_F^{-1}(\mathbf{A}_{0,m}^{(k)}, t_m^{(k)}) \mathbf{F}(\mathbf{A}_{0,m}^{(k)}, t_m^{(k)})$$

where $\mathbf{J}(\mathbf{A}_{0,m}^{(k)}, t_m^{(k)})$ is the Jacobi-Matrix of the vector function $\mathbf{F}$ at $(\mathbf{A}_{0,m}^{(k)}, t_m^{(k)})$.
Irrigation control: towards a new solution of an old problem

The final equations for the Newton iteration scheme read:

\[ J = \begin{pmatrix} \frac{\partial f_1}{\partial A_{0,m}} & \frac{\partial f_1}{\partial t_m} \\ \frac{\partial f_2}{\partial A_{0,m}} & \frac{\partial f_2}{\partial t_m} \end{pmatrix} \] (132)

The final equations for the Newton iteration scheme read:

\[ A_{0,m}^{(k+1)} = A_{0,m}^{(k)} - \frac{f_1 \frac{\partial f_2}{\partial t_m} - f_2 \frac{\partial f_1}{\partial t_m}}{\text{det} J} (A_{0,m}^{(k)}, t_m^{(k)}) \] (133)

\[ t_m^{(k+1)} = t_m^{(k)} - \frac{f_2 \frac{\partial f_1}{\partial A_{0,m}} - f_1 \frac{\partial f_2}{\partial A_{0,m}}}{\text{det} J} (A_{0,m}^{(k)}, t_m^{(k)}) \] (134)

where

\[ \text{det} J = \frac{\frac{\partial f_1}{\partial A_{0,m}} \frac{\partial f_2}{\partial t_m} - \frac{\partial f_2}{\partial A_{0,m}} \frac{\partial f_1}{\partial t_m}}{\frac{\partial f_1}{\partial A_{0,m}} \frac{\partial f_2}{\partial t_m} - \frac{\partial f_2}{\partial A_{0,m}} \frac{\partial f_1}{\partial t_m}} \] (135)

The partial derivatives of the functions \( f_1 \) and \( f_2 \) derived from Equations 129 and 130 are

\[ \frac{\partial f_1}{\partial A_{0,m}} (A_{0,m}, t_m) = \frac{2 \bar{v}(t_m)}{K^2} \frac{\partial \bar{v}}{\partial A_{0,m}} - C_3 \mu_s A_{0,m}^{(k,-1)} - C_2 \cdot \kappa_s \cdot A_{0,m}^{(k,-1)} \] (136)

\[ \frac{\partial f_1}{\partial t_m} (A_{0,m}, t_m) = -x_m + 1 \] (137)

\[ \frac{\partial f_2}{\partial t_m} (A_{0,m}, t_m) = \frac{\partial V_{\text{inf}}(t_m)}{\partial t_m} \] (139)

where \( \frac{\partial \bar{v}}{\partial A_{0,m}} \) and \( \frac{\partial \bar{v}}{\partial t_m} \) have to be deduced from Equation 124.

Exact time derivatives of \( V_{\text{inf}}(t_m) \) are not available from the HYDRUS-2D infiltration model. Consequently, the derivatives have to be approximated by difference quotients which, however, can be afflicted with relatively large errors. Additionally, both the velocity \( v_{\text{tip}}(t_m) \) at the wave tip and the velocity \( v(t_m) \) at the momentum-representative cross section are dependent on \( V_{\text{inf}}(t_m) \) (cf. Equations 124 and 125). Thus, the time derivatives of \( v_{\text{tip}}(t_m) \) and \( v(t_m) \) require a careful approximation as seen later on in this subsection.
Methods: the modules of GAIN-P

Synchronization of surface and subsurface simulation time

Advance time intervals \([t(x_m), t(x_{m+1})]\) (i.e., the advance time between successive \(x_{inf}\)-locations) can reach the order of up to \(10^4\) seconds or even more. This is far too large to be used as the HYDRUS-2D time step. To avoid/minimize numerical errors and to ensure a valid infiltration calculation by the Richards equation, a time-step control is developed which synchronizes the simulation time of HYDRUS-2D with the surface module. The methodology allows HYDRUS-2D to use its own time-step control within the given advance time intervals. To speed up calculations, an adaptive time-step control is applied for increasing/decreasing the HYDRUS-2D time steps for quickly/slowly converging iterations.

Impact of FAPS model assumptions

The main assumption of the analytic solution of the advance model, Equations 55 and 56, is to represent the momentum of the surface water body by the momentum at its moving centre of gravity \(\bar{x}\) (cf. Schmitz and Seus [1990]). Subsequently, the impact of moving the momentum representative cross section \(\bar{x}\) to the entrance section \(x = 0\) is investigated in order to analyze the performance of this numerically even more efficient option. Furthermore, we research the impact of two different approximations and a simple distance-over-time approach on the calculation of wave-tip velocity \(v_{tip}\) on advance. Some remarks on the scaling of the nonlinear system are included in the subsection. Finally, the main aspects involved in constructing the appropriate approximation of \(\frac{\partial V_{inf}}{\partial t}\), which is necessary for implementing the Newton method, are discussed.

Momentum representative cross section

For the case that the momentum described by the right-hand side of Equation 56 refers to the entrance section \(x = 0\) rather than to the moving centre of gravity of the surface water body

\[
\bar{x}(t) = x_m/(\alpha + 2)
\]  

(140)
a modified and simpler version of the new advance model, Equations 129 and 130, can be obtained. Similar to the approach of Schmitz [1989], Equation 124 can then be replaced by the formula

\[
\bar{v}(t_m) = v_0 = Q_0(t_m)/A_{0,m}
\]  

(141)
and the derivatives of Equation 141 are substituted by

\[
\frac{\partial \bar{v}}{\partial A_{0,m}} = -\frac{Q_0(t)}{A_{0,m}^2} \text{ and } \frac{\partial \bar{v}}{\partial t_m} = 0
\]  

(142)
The parameters \(Q(t_m), \bar{A}(t_m)\) and \(\bar{R}(t_m)\) are replaced by \(q(x = 0, t_m), R_{0,m} = p_s A_{0,m}^{p_s}\) and the term \((\alpha_s + 1)/\alpha_s + 2\) is replaced by the factor 1 in the abbreviations \(C_2\) and \(C_3\), so that

\[
C_2 = S_0 \frac{\alpha_s^4}{p_s^3} \text{ and } C_3 = \frac{q_0(t_m)}{g} \frac{\alpha_s^4}{p_s^3}
\]  

(143)
This simplification of Equation 124 and its derivatives is the main advantage of this approach. Because moving the momentum representative cross section to the entrance section on irrigation advance has a substantial impact (cf. paragraph 3.3.5.5), the approach of the moving
centre of gravity of the surface water body is confirmed and subsequently used in the new model FAPS.

Wave-tip velocity
In ZI modelling, the velocity at the wave \( v_{wp}(t) \) is usually approximated by a volume balance approach in conjunction with the assumption of a specific shape of the irrigation wave (e.g. parabolic or power law). Equation 125 also follows this approach. Since this equation is mathematically rather complex, the impact of using simplified equations on calculated velocity is analyzed. Three approximations of \( v_{wp}(t_m) \) are introduced:

i) Full equation 125 as described in Schmitz and Seus [1992]

ii) Simplification of Equation 125 by neglecting terms which are multiplied by \( \partial A_0 / \partial t \) because these addends are small:

\[
v_{wp}(t_m) = (\alpha_s + 1) \frac{Q_0(t_m) - \int_0^{t_m} q(\hat{x}, t_m) d\hat{x}}{A_{0,m}}
\]

(144)

iii) An appropriate difference quotient. Here, velocity is approximated by the change in distance, divided by the change in time:

\[
v_{wp}(t_m) = \frac{\partial x_m}{\partial t_m} \approx \frac{x_m - x_{m-1}}{t_m - t_{m-1}}
\]

(145)

In order to illustrate the impact of these simplifications, \( v_{wp}(t) \) values calculated for the example Cou-A are plotted in Figure 28 at different advance times.

![Figure 28 Realization of wave-tip velocity](image)

It should be noted that Equation 125 is also not an exact formula for calculating \( v_{wp}(t) \). Additionally, for applying Equation 125 in the new model, the time derivative of \( A_{0,m} \), which is supposed to be small, has to be approximated by a difference quotient. Furthermore, an
initial value for the first time interval is needed, which is approximated by $v(t_m) = Q_0(t_m)/A_{0,m}$.\(^{11}\)

Equation 144 is a simplification of Equation 125 by neglecting the second addend in the second bracket of Equation 125. As seen in Figure 28, the impact of the neglected term is slight: the difference between $v(t_m)$ values calculated by Equation 125 and by approximation ii) is about 8% at the end of the first time interval and less than 4% for $t_m > 1000$ s. This impact, however, can be considered minor compared to the approximation error of differentiation. Since the difference quotient Equation 145 yields almost identical values as compared to Equation 144 – and the impact on calculated advance times is negligible, too – Equation 145 is subsequently used in the coupled advance model to calculate $v(t_m)$. Inserting Equation 145 into Equation 124 and differentiating with respect to $A_0$ and $t_m$ yields:

\[
\frac{\partial v}{\partial A_{0,m}} = -\frac{v(t_m) - v(t_m)}{A_{0,m}(\alpha_s + 2)}
\]  

and

\[
\frac{\partial v}{\partial t_m} \approx \frac{1}{(\alpha_s + 2)} \frac{\partial v(t_m)}{\partial t_m}
\]  

where an approximation of $\partial v/\partial t_m$ is obtained by differentiating only the first addend of Equation 124.

Range of functions in the Newton scheme
The nonlinear functions $f_1$ and $f_2$, i.e. the Equations 129 and 130, are solved by the Newton iteration scheme. Values of these functions can lie up to eight orders of magnitude apart, which is unfavourable in terms of a desired fast convergence of the Newton iteration scheme. Normalization of the functions $f_1$ and $f_2$ is discarded because otherwise the normalized system of equations becomes mathematically too complex. Another possibility for reducing the order of magnitude between $A_{0,m}$ and $t_m$ is scaling with an adequate factor. Two approaches are analyzed subsequently: dynamic scaling with the factor $(t_m - t_m)^2$ and constant scaling by multiplication with the squared roughness coefficient $K_{st}^2$. Both types of scaling succeed in narrowing the gap in the order of magnitudes between the values of $f_1$ and $f_2$.

Dynamic scaling
Equation 121 is multiplied by the scaling factor $(t_m - t_m)^2$ to avoid the difference $t_m - t_{m-1}$ in the denominator of Equation 125, which is squared by the differentiation of Equation 121. The focus of this scaling approach is twofold: on the one hand, the representation of the derivatives of $f_1$ is simplified. On the other hand, the convergence and stability of the iteration scheme increases.

In order to illustrate the effect of this scaling, Equation 124 is changed to:

\[
\frac{\partial v}{\partial t_m} \approx \frac{1}{(\alpha_s + 2)} \frac{\partial v(t_m)}{\partial t_m}
\]  

\(^{11}\) For the first time interval $t_1$, both terms in the bracket of Equation 125 are of about the same size, resulting in $v(t_1) \approx 0$. This does not correspond to reality because flow velocity is highest for small simulation times.
This method also contains a dynamic effect: both \( t_m \) and \( t_m - t_{m-1} \) increase during the course of the simulation. Consequently, the scaling factor also increases when the difference between the values of \( A_{0,m} \) and \( t_m \) is similarly on the increase. As a result, the gap between the \( f_1 \) and \( f_2 \) values is now only about two orders of magnitude.

**Constant scaling**

Similar to the procedure described above, Equation 129 and its derivatives Equation 136 and 137 are multiplied by the constant scaling factor \( K^2_{st} \). As a consequence, the values of \( f_1 \) and \( f_2 \) are about three orders of magnitude apart, which corresponds to an improvement of five orders.

Since the presented solution method is already highly convergent, the overall benefit of scaling to faster convergence is very slight. For this reason we stress that the appropriate derivative approximation of the volume balance equation is much more essential to numerical stability as described in the following subsection.

**Time derivation of the volume-balance equation**

The exact time derivative of the volume balance, Equation 130, is not easily accessible. In a first attempt, the second term of Equation 139 is replaced by a difference quotient, i.e. the actual value of \( V_{inf} \) and the value from the previous time step divided by the time interval. It is the slope of total infiltrated water volume over time:

\[
\frac{\partial V_{inf}(t_m)}{\partial t_m} \approx \frac{V_{inf}(t_m) - V_{inf}(t_{m-1})}{t_m - t_{m-1}} \quad (149)
\]

Especially for small infiltration opportunity times and large time intervals, \( t_m - t_{m-1} \), this first approximation (i) is rather poor. To illustrate this point, we again take example Cou-A (cf. 3.3.5.2): FAPS is applied in order to obtain values of \( t_m \) and \( A_{0,m} \) at the locations \( x_i = 32.5 \) and 65.0 m.\(^{12}\) The function \( f_2(t_m, \hat{A}_{0,m}) = \hat{f}_2(t_m) \) is considered for fixed \( \hat{A}_{0,m} \) and the corresponding derivative \( \partial f_2 / \partial t_m \) approximation i) at \( \hat{A}_{0,m} \).\(^{13}\) The function \( \hat{f}_2 \), i.e. Equation 130 and its time derivative approximated by Equation 149 are plotted over time in Figures 29a) and b), respectively.

\(^{12}\) Constant scaling by \( K^2_{st} \) is applied for the presented example run (cf. subsection 3.3.5.3).

\(^{13}\) For the given example, it is \( A_{0,m} = 0.00449 \text{ m}^2 \) and \( A_{0,m} = 0.00465 \text{ m}^2 \) for the locations \( x_i = 32.5 \) and 65.0 m, respectively.
Methods: the modules of GAIN-P

Graphically, the solution of the example problem is found where \( \hat{f}_2 \) meets zero, i.e. at \( t_m \approx 280 \text{ s} \) and \( t_m \approx 900 \text{ s} \) for the locations \( x_i = 32.5 \text{ and } 65.0 \text{ m} \), respectively. Contrary to the observed slope of the \( \hat{f}_2 \) function for location \( x_i = 65.0 \text{ m} \) (3rd iteration loop), the negative values of the derivatives suggest a decreasing shape of \( \hat{f}_2 \) for \( t_{m} < 600 \text{ s} \) and a local minimum at about 600 s. It can be shown that the derivative is negative for a wide range of simulation scenarios with small irrigation advance rates. Consequently, wrong iterative or non-convergence of the Newton method has to be expected, which finally results in erroneous advance times and/or model termination.

For this reason, an adaptive control of the time step \( \Delta t_{inf} \) is implemented in FAPS, which satisfactorily represents the time derivation \( \partial V_{inf}(t_m)/\partial t_m \):

\[
\frac{\partial V_{inf}(t_m)}{\partial t_m} \approx \frac{V_{inf}(t_m) - V_{inf}(t_m - \Delta t_{inf})}{\Delta t_{inf}}
\]

(150)

It should be noted that no extra computation is required to determine \( V_{inf}(t_m - \Delta t_{inf}) \).

Applying this difference approximation technique (approximation ii), the Newton iteration scheme converges to the correct solution as seen in Figure 30. In this figure, the iteration steps of the third iteration loop, i.e. the search of \( A_{0,m} \) and \( t_m \) at location \( x_i = 65.0 \text{ m} \), are tracked in the \( A_0 - t \) plane with the contour plots of the functions \( f_1 \) and \( f_2 \) (Equations 129 and 130). The ordinate (time axis) of the right-hand plot of Figure 30 is given in close-up detail. The

Figure 29  Time derivation of the FAPS volume-balance function
solution of this particular iteration is found where the zero contour lines of both functions $f_1$ and $f_2$ meet.

The blue circles in Figure 30 represent the subsequent iteration steps, using approximation i). Large advance times, both positive and negative (!), are obtained. The scheme does not converge and, moreover, $t_m$ values oscillate from the 6th iteration onwards. In contrast, the scheme is highly convergent when approximation ii) is used in combination with $\Delta t_{inf} = 0.9 \left( t_m - t_{m-1} \right)$ as seen by the red circles in Figure 30. The calculated $t_m$ value of the third iteration is very close to the correct value of $t_m = 902.4$ s for the advance time at $x_{inf} = 65$ m. The $A_0$ value, however, is always calculated to a value close to the solution, $A_{0,m} = 4.658 \cdot 10^{-3} \text{ m}^2$, independent of the approximation used.

### 3.3.5.4 Coupling during storage, depletion and recession phases

In contrast to the advance model, the surface flow during the storage phase (S-II) as well as during the depletion and recession phase is assumed to be uniform (cf. subsection 3.3.2). The surface and subsurface flow equations of these phases are coupled by the continuity equations 55 and 98, respectively. The necessary boundary condition for HYDRUS-2D is provided by Equation 80, i.e. the mean flow depth $\overline{h}_{inf}$. This flow depth accounts for $0 \leq x \leq x_L$ during S-II and for $x_u \leq x \leq x_L$ (section II) during the depletion and recession phase (cf. subsection 3.3.2.3). According to the assumptions made for the depletion and recession model, the horizontal water level in section (I), $h^{(I)}_{inf}(x, t)$, is calculated by linear interpolation between the flow depth at $x_{tail}$ and $x_n$:

$$h^{(I)}_{inf}(x, t) = h^{(II)}_{inf} \frac{x - x_{tail}(t)}{x_n(t) - x_{tail}(t)}$$

(151)
For both the depletion and the recession phase, the water level \( h_{wl}^{(t)}(x, t) \) is zero for all \( x \leq x_{tail} \) in section (I).

### 3.3.5.5 FAPS model test

A comprehensive sensitivity analysis of the FAPS model was conducted. It investigated the impact of various input parameters (field length, furrow cross section, field slope, channel roughness, soil-hydraulic parameters, initial soil water content, etc.) on the model results [Kuiry, 2002, Reuter, 2003]. In addition, the impact of the time interval length and space discretization on simulation results was analyzed by Wöhling et al. [2004b]. The contribution also discusses a possibility for reducing the number of HYDRUS-2D calculations by an interpolation scheme of cumulative infiltration.

During various tests which are subsequently discussed, the new solution FAPS showed a much better convergence behaviour and an improved numerical stability when compared to the fixpoint approach FAP. FAPS also employs small inflow rates which would inevitably lead to FAP model abortion. Irrigation advance calculated by both FAPS and FAP is identical for convergent runs with high inflow rates. The space-discretized model, however, requires considerably less iterations and thus less computation time.

Further tests in this subsection confirmed the choice of the momentum representative cross section at the centre of gravity of the surface water body.

#### Comparative analysis of Newton's scheme versus fixpoint iteration

The performance of the new solution strategy FAPS, i.e. Newton's method for predetermined space increments, is compared to FAP, i.e. fixpoint iteration, in combination with a time discretization. A time step of \( \Delta t = 180 \text{ s} \) is applied to the fixpoint iteration scheme.

The simulation of the Cou-A (Run 1) irrigation advance by FAPS is highly convergent and only requires a total of 15 iterations in four iteration loops for calculating the total advance time (an average of 3.8 iterations per iteration loop). In contrast, the fixpoint scheme FAP requires 393 iterations. Moreover, the advance curve of the FAP simulation tends towards a constant advance rate which is not realistic. In contrast, the predicted advance times of the FAPS model compare perfectly to observations as shown in subsection 4.1.2.

In order to give a more general picture of the convergence behaviour and range of applicability of the two approaches, altogether 34 simulations of the numerical experiment Cou-A (described above) were conducted by varying the inflow rate from \( Q_0(t) = 0.0001 \text{ m}^3/\text{s} \) to \( Q_0(t) = 0.0034 \text{ m}^3/\text{s} \) in steps of \( \Delta Q_0 = 0.0001 \text{ m}^3/\text{s} \). Figure 31 shows the total number of iterations of the simulations with both FAPS and FAP in relation to the inflow rate. The new solution FAPS shows a fast convergence for all test runs, i.e. also for very low inflow rates, which correspond to a low irrigation advance rate. Iteration numbers, i.e convergence behaviour, are practically invariant to the inflow rate for \( Q_0 \geq 0.0004 \text{ m}^3/\text{s} \). The average total number of iterations for calculating irrigation advance is 15.2, which corresponds to 3.8 iterations per iteration loop.

In contrast, the iteration numbers of the fixpoint iteration FAP are much higher as seen in Figure 31. The average iteration number is 96.4, which is very high compared to FAPS. For \( Q_0 < 0.0021 \text{ m}^3/\text{s} \), the solution of the test runs does not always converge as indicated by the gaps in the graph of Figure 31. Below \( Q_0 < 0.001 \text{ m}^3/\text{s} \), FAP does not yield a convergent solution at all. It should be noted that the chosen time step of 180 s is relatively large for a
convergent solution of the Richards equation (HYDRUS-2D). A reduction of the time step, however, does not improve convergence, yet it does increase the iteration number and, thus, also the computation effort.

This comparative study, furthermore, illustrates the FAPS superiority over FAP in both computational efficiency as well as in its more extensive range of resolvable inflow rates.

**Impact of momentum at field inlet**
In order to analyze the impact of the assumption that the momentum is referred to the entrance section \( x = 0 \) rather than to the moving centre of gravity of the surface water body (subsection 3.3.5.3), comparative simulations are conducted using the following example:

**Example Cou-B:** The furrow geometry, slope, roughness and spacing of infiltration sections is taken as in Cou-A. The inflow rate is \( Q_0 = 0.002 \text{ m/s} \). Infiltration characteristics are described by the Kostiakov parameters: \( k_k = 6.4 \cdot 10^{-4} \text{ m}^2\text{s}^{-1} \), \( a_k = 0.4 \), \( c_k = 5.0 \cdot 10^{-7} \text{ m}^2\text{s}^{-1} \).

The following terminology is introduced to distinguish the model versions:

**FAPS/FAP-xquer:** FAP/FAPS model with representative cross section at the moving centre of gravity.

**FAPS/FAP-x0:** FAP/FAPS model with representative cross section at the entrance section.

Figure 32 shows the irrigation advance times resulting from simulations of the test run Cou-B with the momentum representative cross section chosen at the moving centre of gravity of the surface water body (FAP/S-xquer) and at the furrow inlet (FAP/S-x0), respectively. The difference between the simulated total advance time of FAPS-x0 and FAP-x0 is almost indistinguishable with 0.3 s (0.03%). This homogeneity in model results shows the consistency of the transformation from time to space discretization and, furthermore, that different solution methodologies lead to the same results, as should be expected. Between the FAPS-xquer and the FAP-xquer advance times, however, there is a small difference of 11.9 s (1.1%) which results from the small relaxation parameter \( \text{relx} \), which had been adjusted during the FAP-xquer run (cf. subsection 3.3.5.2).
Figure 32 Simulated advance times by FAPS and FAP with (i) momentum representative cross section at the moving centre of gravity of the surface water body and (ii) at the field inlet

The total advance time for the simulation by FAPS-x0 deviates significantly from that by FAPS-xquer by -84.2 s (-8.4%). This (systematic) faster advance rate of FAPS-x0 is caused by decreasing values of the function $f_1$, when the parameters $\bar{q}, \bar{q}$ related to the moving centre of gravity in Equation 129 are replaced by the corresponding parameter at the field inlet (as a consequence of $A_0 > \bar{A}$). Since the FAPS-x0 deviation in calculated advance is significant, the approach of the momentum representative cross section at the moving centre of gravity of the surface water body $\bar{x}$ is confirmed for use in the new model FAPS.

### 3.3.5.6 Coupling surface-subsurface flow: summary

A comprehensive furrow irrigation model for single irrigation events is developed in the preceding subsection and its performance is analyzed and validated on the basis of various laboratory and field experiments. The model consists of the analytical zero-inertia surface irrigation model FAPS and the physically based two-dimensional infiltration model HYDRUS-2D, which are coupled by an iterative procedure (FAPS-H). A second version of the model is developed by coupling FAPS with the empirical Kostiakov formula (FAPS-K) in order to compare simulation results with the results of other irrigation models. It is shown that solving the set of nonlinear differential equations for predetermined space increments by Newton's iteration method is far more efficient, highly convergent, more accurate and numerically stable than the solution for specified time-increments by the fixpoint iteration method.

FAPS-H can predict the surface flow with high accuracy and provides distributed infiltration data during all the phases of irrigation. In addition, it gives a quasi-3D picture of the transient soil water transport. Some of the irrigation performance criteria, such as the distribution uniformity, application efficiency, etc., can be calculated with much more precision as compared to the volume balance approaches commonly used in practice.

In furrow irrigation modelling, an accurate estimation of irrigation advance requires an appropriate evaluation of the infiltration process. This, at first glance, seems to imply a high resolution of infiltration computations along the furrow, i.e. a large number of numerical infiltration models along the furrow reach, which can lead to very large CPU-time

---

14 Similarly, FAP-x0 total advance time deviates from that of the FAP-xquer run.
requirements. However, the preceding study revealed that the irrigation model is less sensitive to the number of HYDRUS-2D models, i.e. the length of the sub-reaches $\Delta x_{\text{inf}}$, than one would have expected. In this context, it turned out that accurate estimations of infiltration from the entire wetted furrow can be achieved by employing even a coarse resolution of calculation points along the furrow. As confirmed by model simulations, the length of the sub-reaches can be taken as $\Delta x_{\text{inf}} \leq 0.25 \cdot x_L$ with only a negligible loss of accuracy regarding both cumulative infiltration $V_{\text{inf}}(t)$ and irrigation advance time $t_s$. FAPS calculates advance times rather than advance distance and, thus, does not require the choice of a time interval $\Delta t$, which has significant impact on FAP model results (Wöhling et al., 2004b).

### 3.3.6 Coupling subsurface flow and crop growth

Crop modelling in the present context comprises the simulation of crop development in the field during a whole growing period. On the macroscopic scale, a soil-plant-atmosphere continuum is assumed for daily averages, i.e. water extracted by the plant roots is completely transpired to the atmosphere (cf. subsection 3.3.1.4). By this assumption, the potential root water uptake is equal to the potential transpiration TP. In the presented modelling approach, the actual root water extraction (= actual transpiration TA) depends on the availability of soil water in the root zone, which is indicated by the water-stress factor stress. This factor also influences the crop development, i.e. the development of the LAI, which in turn has an impact on the potential transpiration. These mutual dependencies are visible in Figure 33, which shows the principles of feed-back coupling of the soil-plant-atmosphere system.

The crop model LAI-SIM comprises the LAI simulation, the calculation of the splitting coefficient $c_p$, potential evaporation and transpiration, as well as root growth (cf. Figure 33). In this section, (i) the common boundaries between LAI-SIM and HYDRUS-2D are defined, (ii) the calculation of the water-stress factor, stress, is outlined, (iii) the coupling method LAI-SIM $\rightarrow$ HYDRUS-2D is described, and (iv) a sensitivity analysis of the LAI-SIM input

---

**Figure 33** Principles of coupling the crop model LAI-SIM and the subsurface flow model HYDRUS-2D
parameters with respect to predicted LAI is conducted on the basis of the 1999 experiments on corn at the Lavalette plot (France).

### 3.3.6.1 Common boundaries

Basically, there are two different common boundaries between the crop model LAI-SIM and the subsurface flow model HYDRUS-2D. The first boundary is the soil-atmosphere interface which is associated with the processes of evaporation and precipitation as seen in Figure 34a). The second boundary consists of the internal calculation nodes of the HYDRUS-2D calculation mesh which are associated with the root water uptake by plants as seen in Figure 34b).

![Figure 34 Common boundaries LAI-SIM HYDRUS-2D](image)

#### Soil-atmosphere boundary

The soil-atmosphere boundary can be both a flux and a gradient type boundary. Nodal values of the net flux \( q_{at}^{(node)}(t) \) across the boundary (evaporation minus precipitation) are calculated by

\[
q_{at}^{(node)}(t) = \frac{EP(t) - P(t)}{fs/lm} \cdot \frac{l_{surf}}{l_{at}^{(node)}}
\]

with daily mean EP and TP-values in \([\text{m}^3 \text{m}^{-2} \text{s}^{-1}]\) and the boundary length associated to the node \( l_{at}^{(node)} \) in \([\text{m}]\).

The net flux across the atmospheric boundary is calculated by HYDRUS-2D, which is positive when directed out of the soil. In Equation 152, the net flux is scaled both by the width of the furrow \( fs \) (furrow spacing) and by the actual length of the boundary \( l_{surf} \) (cf. Figure 34a).
It should be noted that the furrow spacing is smaller than the surface length during redistribution times $fs < l_{surf}$ due to the curvature of the furrow. At times of irrigation, however, $l_{surf}$ changes with time according to the rising and falling flow depth in the furrow. Consequently, the boundary type of the individual furrow calculation nodes must be determined and set to either flux or constant pressure type before each HYDRUS-2D time step $\Delta t$ in accordance with the flow depths calculated by the surface model. If evaporation has caused the matric head of an atmospheric flux-type boundary node to fall below the limit $h_{crit}$ the boundary type of this node is changed to the prescribed head type. The value of $h_{crit}$ is determined by the equilibrium conditions between soil water and atmospheric vapour [Simůnek et al., 1996].

Root water uptake

The second common boundary of crop and subsurface flow modelling is the number of internal HYDRUS-2D calculation nodes which are located within the root zone. A flux is assigned to these nodes, which is associated with the plant root water extraction as seen in Figure 34b).

The shape, activity and development of the plant root system specifically depends on the crop type but also on environmental factors such as the soil type and the soil water availability. Rooting depths of different crops have been observed for different growing stages and are reported in detail in literature (e.g. Achtinhich [1980], Doorenbos and Kassam [1986]). A simple and frequently used modelling approach for root development is the assumption of linear growth during the initial and crop development stage – until a maximum root depth $z_{r,max}$ is reached. After that, no further development is assumed during mid-season and late-season until the harvest takes place (see Figure 35).

Figure 35  
Crop coefficient $K_c$ and root depth $z_r$ during a growing season

Other assumptions made in this modelling approach are:

- The roots develop uniformly across the total width of the area ($fs$).
- The root zone expansion depends only on time.
Water stress during the growing period is of no consequence for root development since plant roots are assumed to advance deeper into the soil, even if the plant itself is under water stress. As a result, the root depth $z_r$ depends on time only. The parameters $z_{r,\text{init}}$, $z_{r,\text{max}}$ and $t(z_{r,\text{max}})$ denote initial root depth, maximum root depth and the time of maximum root depth, respectively.

Generally, it is possible to introduce the development of an arbitrarily shaped root zone within the y-z plane in HYDRUS-2D. However, parameterization effort increases greatly. This is because algorithms must be developed for identifying, at any given time of the simulation, the affiliation of each HYDRUS-2D calculation node to the root zone by matching it with the simulated root zone geometry. For common crops in furrow irrigation this effort is simply not necessary because the assumptions made above are reasonably close to reality.

**Root activity**

The degree in which different parts of the plant rooting system play a role in soil water extraction is referred to as ‘root activity’. HYDRUS-2D simulates root activity by weighting nodal fluxes within the root zone according to their activity index. This is realized by a normalized distribution of water extraction $b(y, z)$ [Simůnek et al., 1996]:

$$b(y, z) = \frac{b'(y, z)}{\int_{\Omega_R} b'(y, z) d\Omega}$$ (153)

where $\Omega_R$ and $b'(y, z)$ denote the (arbitrarily shaped) root zone domain and the arbitrary distribution of water extraction activity (to be predetermined by the model user), respectively (cf. Figures 34b) and c)). The normalization ensures that the integral of $b(y, z)$ over the total root domain $\Omega_R$ is always unity:

$$\int_{\Omega_R} b(y, z) d\Omega = 1$$ (154)

The assumption that the root activity is only dependent on soil depth reduces Equation 153 to

$$b(z) = \frac{b'(z)}{\int_{\Omega_R} b'(z) d\Omega}$$ (155)

As seen in Figure 34c), the arbitrary distribution function $b'(z)$ is now described by the exponential function of Novak [1987], which has been experimentally confirmed:

$$b'(z) = \alpha_N \cdot \exp(-\alpha_N \cdot z/z_*)$$ (156)

where $\alpha_N$ is an empirical constant usually of order $10^6$ for well-developed root systems and leaf canopies [Kutilek and Nielsen, 1994]. An option of uniform root activity is also included in the presented model approach, as indicated in Figure 34c).

**3.3.6.2 Plant water uses**

To a crop, stress means the divergence of environmental factors from optimal growth conditions. This can be caused by chemicals (salt) and/or by either too little or too much...
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Water in the root zone, which is referred to as 'water stress'. In the present study only water stress is considered. For the case that the environmental conditions of the crop are at an optimum, the actual and potential transpiration are equal, i.e. \( TA = TP \). Water stress reduces root water uptake and, consequently, \( TA < TP \). This phenomenon is described by

\[
TA = s(h_m)TP
\]

(157)

where the water stress response function \( s(h_m) \) by Feddes et al. [1978] is a prescribed dimensionless function (Figure 36) of the soil water pressure head \( h_m \):

\[
\begin{align*}
0 & \quad \text{for} \quad h_m > h_0 \text{ and } h_wp \geq h_m \\
\frac{h_m - h_{opt}}{h_0 - h_{opt}} & \quad \text{for} \quad h_0 > h_m > h_{opt} \\
1 & \quad \text{for} \quad h_{opt} \geq h_m > h_d \\
\frac{h_m - h_wp}{h_d - h_wp} & \quad \text{for} \quad h_d \geq h_m > h_wp
\end{align*}
\]

(158)

Figure 36 Water stress response function \( s(h_m) \)

The root water uptake is assumed to be zero for a soil water content close to saturation, i.e. wetter than some arbitrary 'anaerobiosis point' \( h_0 \). The water uptake is also zero for matric heads less than the wilting point \( h_wp \). Water uptake is considered optimal between the matric heads \( h_{opt} \) and \( h_d \), whereas for matric heads between \( h_d \) and \( h_wp / h_0 \) and \( h_{opt} \) water uptake decreases/increases linearly with the matric head as seen in Figure 36. The matric head \( h_d \) is associated with the turgor pressure below which the plant starts closing the stomata.

The HYDRUS-2D code permits the user to make the variable \( h_d \) a function of the potential transpiration rate \( TP \) (\( h_d \) presumably decreases at higher transpiration rates) as indicated in Figure 36. It currently employs a linear interpolation scheme [Simunek et al., 1996], where \( h_{d(1)} \) and \( h_{d(2)} \) are critical turgor pressure values associated with the potential transpiration rate \( TP(h_{d(1)}) \) and \( TP(h_{d(2)}) \) respectively, considering the constraint \( TP(h_{d(1)}) < TP(h_{d(2)}) \).
The actual root water uptake, i.e. the actual transpiration \( TA^{(i)}(t) \), is calculated for each cross section \((i)\) along the furrow and for each time interval. Daily cumulative actual transpiration \( TA^{(i)}(\text{das}) \) is calculated by

\[
TA^{(i)}(\text{das}) = \sum_{t=(\text{das}-1)\cdot86400}^{\text{das}\cdot86400} TA^{(i)}(t) \cdot \Delta t
\]

(159)

where \( \Delta t \) denotes the HYDRUS-2 time step. Finally, the ratio between the actual and the potential daily transpiration of any given simulation day \((\text{das})\) leads to the water stress index which is associated with the next simulation day \((\text{das}+1)\):

\[
\text{stress}^{(i)}(\text{das}+1) = \frac{TA^{(i)}(\text{das})}{TP^{(i)}(\text{das})}
\]

(160)

where \( 0 \leq \text{stress}^{(i)} \leq 1 \). It should be noted that when \( \text{stress} = 1 \) the crop has transpired throughout the whole day with the potential rate. In contrast, the actual transpiration has been zero during the same time interval when \( \text{stress} = 0 \). This stress index is used in Equation 108 to calculate the daily values of leaf area index \( \text{LAI}^{(i)}(\text{das}) \) at each predetermined cross section along the furrow.

### 3.3.6.3 Crop model coupling methodology

The index \((i)\) describing the affiliation of parameters to a specific cross section is neglected in this subsection in order to simplify the notation. Equation 160, which is always evaluated at 0:00:00 a.m. simulation time, provides the mean water stress index of the previous 24 hours. In order to avoid unacceptably long calculation times, the stress-value of the simulation day \((\text{das})\) is assumed to be a reasonable estimate for the next simulation day \((\text{das} + 1)\) rather than applying an iterative coupling method to the crop-subsurface interaction. The resulting coupling scheme is presented in Figure 37. After initialization of the variables \((t, \text{LAI}, K_c, ET_0 ..)\) and the calculation of daily TP and EP values by Equations 104 and 105, HYDRUS-2D simulations are executed in a loop until the simulation time is equal to the end of the day \((\text{das})\), where \( \text{das} = 1..\text{nsd} \) denotes the number of the days after sowing. Daily cumulative values of \( TA(\text{das}) \) are calculated by Equation 159 within this loop. At the end of a simulation day, the stress index \( \text{stress}(\text{das} + 1) \) is calculated by Equation 160. Now, the simulations advance to the next day \((\text{das} = \text{das} + 1)\) with the calculation of \( \text{LAI} \) and \( K_c \) values given by Equations 108 and 107, respectively.

Apart from the prescribed coupling approach, the broken line boxes and arrows in Figure 37 indicate the option of an iterative coupling. Here, the calculated \( \text{stress}_{(j=1)} \) is used (in addition to the approach described above) for correcting the potential values of evaporation and transpiration for the present calculation day until the deviation between the actual and previous stress-index falls below a certain limit:

\[
|\text{stress}_{(j)} - \text{stress}_{(j-1)}| < \varepsilon
\]

(161)

where \( j \) denotes the iteration count. This iterative coupling is especially time-consuming because the HYDRUS-2D calculations of each simulation day have to be repeated 2-5 times. It has to be borne in mind that HYDRUS-2D calculations take up more than 90% of the total
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computing time. However, it is not necessary to iteratively couple LAI-SIM and HYDRUS-2D as is shown subsequently.

Stress-index prediction
The impact of predicting the stress index by Equation 160 on crop development (LAI) is subsequently analyzed by imposing a systematic error to the stress index during the whole growing season. By doing this, the stress index is both systematically overestimated and underestimated at a physically possible maximum level. A reference simulation (indicated by index \textit{ref}) is conducted for a total of \textit{nsd} = 132 simulation days using the HYDRUS-2D parameterization of the 1999 irrigation experiments at the Lavalette site (France) (cf. subsection 4.1.1.4) and the LAI-SIM parameterization of corn \textit{Samsara}, cf. Table 6). Since we deliberately omitted the experimental furrow irrigations from our test simulations, the crop is under heavy water stress around the 65\textsuperscript{th} simulation day, following a period of 40 days with almost optimal conditions for plant growth,\textsuperscript{15} as seen in Figure 38a). The soil moisture available in the root zone increases in volume until the 82\textsuperscript{nd} day and drops again to a second depression at 105 simulation days. The onset of plant senescence results in lower plant water demand and a decreasing stress index. The two stress peaks also appear in the simulated

\textsuperscript{15} The rainfall events and the sprinkler irrigation during this period are included in the simulation.
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LAI function in Figure 38b) as was to be expected. The time delay of the peaks is typical for the plant response to water stress. Figure 38b) also shows the potential LAI function for optimum plant water supply, i.e. no water stress.

Based on stress-index values \( \text{stress}_{ref} \) of the reference run, the average stress gradient for subsequent simulation days

\[
\overline{\Delta \text{stress}} = \frac{1}{(\text{nsd} - 1)} \sum_{\text{das}=2}^{\text{nsd}} \left| \text{stress}_{ref}(\text{das}) - \text{stress}_{ref}(\text{das} - 1) \right|
\]

and the maximum stress gradient for subsequent simulation days

\[
\Delta \text{stress}_{\text{max}} = \max \left( \left| \text{stress}_{ref}(\text{das}) - \text{stress}_{ref}(\text{das} + 1) \right| \right)
\]

are calculated to \( \Delta \text{stress} = 0.007 \) and \( \Delta \text{stress}_{\text{max}} = 0.045 \) (4.5% of the total stress range), respectively. The impact of systematically over/underestimating the reference stress index \( \text{stress}_{ref}(\text{das}) \), by both the average and maximum simulated stress-index gradient, is subsequently analyzed by adding/subtracting the values to/from the reference stress index during the simulation. Thus, four runs are conducted which differ from the reference run by the imposed stress index:

\[
\text{stress}(\text{das}) = \text{stress}_{ref}(\text{das})
\]

\[
\text{(1)} \quad +\Delta \text{stress} \left( 1 - \text{stress}_{ref}(\text{das}) \right)
\]

\[
\text{(2)} \quad -\Delta \text{stress} \left( 1 - \text{stress}_{ref}(\text{das}) \right)
\]

\[
\text{(3)} \quad +\Delta \text{stress}_{\text{max}} \left( 1 - \text{stress}_{ref}(\text{das}) \right)
\]

\[
\text{(4)} \quad -\Delta \text{stress}_{\text{max}} \left( 1 - \text{stress}_{ref}(\text{das}) \right)
\]
with the additional conditions:

\[
\text{stress}(\text{das}) = \begin{cases} 
1 & \text{for stress}(\text{das}) > 1 \\
\text{stress}(\text{das}) & \text{for } 0 \leq \text{stress}(\text{das}) \leq 1 \\
0 & \text{for stress}(\text{das}) < 0 
\end{cases}
\] (165)

The stress index functions and the corresponding LAI functions of the four runs are plotted in Figures 38a) and b). Both the stress index and LAI functions of the runs (1) and (2) are almost identical to the reference functions. The maximum LAI differences are calculated to

\[
\Delta\text{LAI}_{\text{max}} = \max \left( \left| \text{LAI}_{\text{ref}}(\text{das}) - \text{LAI}_{(1)/(2)}(\text{das}) \right| \right) = \pm 0.014 \text{ m}^2 / \text{m}^2.
\]

This corresponds to 0.5% of the simulated total LAI range, with the maximum leaf area index \(\text{max} \left( \left| \text{LAI}_{\text{ref}}(\text{das}) \right| \right)\) calculated to 2.96m²/m².

Runs (3) and (4), i.e. the simulations where the maximum simulated stress-index gradient was imposed for each simulation day, do not result in significant differences in predicted LAI values, either. The maximum deviation between the LAI values of the reference run and both runs (3) and (4) is 0.086m²/m², which is about 2.9% of \(\text{max} \left( \left| \text{LAI}_{\text{ref}}(\text{das}) \right| \right)\). It should be noted that runs (3) and (4) are included in the analysis to show the impact of possible stress-index prediction errors in respect of the worst case scenarios, which, in fact, are unlikely to occur. As seen in Figure 38b), the deviations between the LAI values of runs (3)/(4) and the reference run are at their greatest when water stress is at a maximum (i.e. the stress index is at a minimum). However, the deviations decrease again after a heavy stress period (instead of accumulating, which results in an increasing gap between the values of the LAI functions), as the water stress is decreasing, too. It can be concluded that Equation 160 provides a sufficiently accurate estimate of the stress index. For this reason, iterative coupling of the crop model LAI-SIM and HYDRUS-2D is not necessary and is discarded in the presented modelling approach.

### 3.3.6.4 Crop model sensitivity analysis

A LAI-SIM input parameter sensitivity is subsequently analyzed with respect to predicted LAI values. LAI-SIM uses altogether 18 independent input parameters. Eight more parameters are related to root water extraction in the subsurface model HYDRUS-2D and are therefore also included in the sensitivity analysis. The list of all parameters considered in the analysis is given in Table 9. Two control runs were conducted: (i) a run with optimum crop growth conditions (CMS-o) and (ii) a run where the crop is under water stress during the growing season (CMS-s). For the latter case, the experimental run at Lavalette (1999) is used with the same parameterization as in the analysis of stress-index prediction (subsection 4.1.2.2). Optimum crop conditions are adjusted by simply setting the stress index to \(\text{stress}(\text{das}) = 1\) for the entire simulation time. Note that for the CMS-o run only a total of 11 input parameters are relevant, whereas this increases to 26 parameters for the CMS-s run.

LAI-SIM is parameterized for the CMS-o/CMS-s runs according to subsection 3.3.6.3, to Table 6 (input parameters of the Maize). A summary of the plant/soil parameter values is given in Table 9.
Methods: the modules of GAIN-P

Table 9 Parameter values of the LAI-SIM sensitivity analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Control Value</th>
<th>Parameter</th>
<th>Control Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LAI_{max} \ [m^2 \cdot m^{-2}]$</td>
<td>4.5 2.25 6.75</td>
<td>$z_{r,max} \ [m]$</td>
<td>1.2 0.6 1.8</td>
</tr>
<tr>
<td>$dens \ [P \cdot m^{-2}]$</td>
<td>10 5 15</td>
<td>$z_{r,init} \ [m]$</td>
<td>0.1 0.05 0.15</td>
</tr>
<tr>
<td>$T_b \ [\circ C]$</td>
<td>6 3 9</td>
<td>$t (z_{r,max}) \ [d]$</td>
<td>72 36 108</td>
</tr>
<tr>
<td>$T_s \ [C \cdot d]$</td>
<td>100 50 150</td>
<td>Root activity (Novak, 1987)</td>
<td>no yes -</td>
</tr>
<tr>
<td>$T_f \ [C \cdot d]$</td>
<td>1005 503 1508</td>
<td>$h_0 \ [m]$</td>
<td>-0.1 -0.05 -0.15</td>
</tr>
<tr>
<td>$T_{mat} \ [C \cdot d]$</td>
<td>1925 963 2030$^a$</td>
<td>$h_{d(1)} \ [m]$</td>
<td>-3.0 -1.5 -4.5</td>
</tr>
<tr>
<td>$T_{crit1} \ [C \cdot d]$</td>
<td>900 450 1350</td>
<td>$h_{d(2)} \ [m]$</td>
<td>-35.0 -17.5 -52.5</td>
</tr>
<tr>
<td>$T_{crit2} \ [C \cdot d]$</td>
<td>1600 900$^b$ 2030$^c$</td>
<td>$h_{up} \ [mi]$</td>
<td>-160 -80 200$^c$</td>
</tr>
<tr>
<td>$\delta_1 \ [-]$</td>
<td>14 7 21</td>
<td>$h_{opt} \ [m]$</td>
<td>-0.3 -0.15 -0.45</td>
</tr>
<tr>
<td>$\delta_2 \ [-]$</td>
<td>0.2 0.1 0.3</td>
<td>$TP(h_{d(1)}) \ [m m d^{-1}]$</td>
<td>$6.0 \cdot 10^{-8}$ $3.0 \cdot 10^{-8}$ $9.0 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>$\lambda \ [-]$</td>
<td>1.25 0.63 1.88</td>
<td>$TP(h_{d(2)}) \ [m m d^{-1}]$</td>
<td>$1.6 \cdot 10^{-8}$ $0.8 \cdot 10^{-8}$ $2.4 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>$\beta \ [-]$</td>
<td>2.4 1.2 3.6</td>
<td>$h_{critA} \ [m]$</td>
<td>-200 -160$^c$ -300</td>
</tr>
<tr>
<td>$K_{c,max} \ [-]$</td>
<td>1.2 0.6 1.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Maximum thermal time.

$^b$ $T_{crit2} \geq T_{crit1}$

$^c$ $h_{up} \leq h_{critA}$

Similar to the approach described in subsection 4.1.2.2, the sensitivity index SI is calculated for a change in the input parameter values of –50% and +50%. Only one parameter is changed at a time, whereas the other parameters retain the values of the control run. LAI-SIM calculations are conducted and presented subsequently for a single furrow cross section.

Table 10 shows the calculated sensitivity index SI for both control situations. In the case of the CMS-o run, the parameters related to root growth and water stress have no influence on the predicted LAI, as was to be expected. The most sensitive input parameter is $\beta$, due to its position as an exponent in the LAI Equation 108, which is confirmed by an SI value of +7.2%. $\beta$ is a shape parameter which has to be carefully calibrated on the basis of well-watered field experiments. Also highly sensitive to the LAI are threshold thermal time $T_f$, emergence thermal time $T_s$, base thermal time $T_b$ and maximum LAI, which is confirmed by SI values close to or above 1.0 (cf. Table 10). These values can be estimated with relatively high certainty and have often been mentioned in literature (e.g. Achtenich [1980], Mailhol et al. [1997], Mailhol [2001], Neitsch et al. [2002]). The parameters in the analysis of CMS-o have a moderate or low sensitivity to predicted LAI values as seen in Table 10. The LAI-SIM input parameters are sorted in order of decreasing SI in Table 11. Note that the maximum crop coefficient $K_{c,max}$ has no direct impact on LAI calculation, but it does indirectly on the potential transpiration rate TP.
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Table 10  LAI-SIM input parameter sensitivity for LAI

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Lavalette – optimum run</th>
<th>Lavalette – stressed run</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SI – LAI (CMS-o)</td>
<td>SI – LAI (CMS-s)</td>
</tr>
<tr>
<td></td>
<td>-50%</td>
<td>+50%</td>
</tr>
<tr>
<td>$I_{AImax}$</td>
<td>-0.94</td>
<td>0.93</td>
</tr>
<tr>
<td>$T_f$</td>
<td>1.36</td>
<td>1.64</td>
</tr>
<tr>
<td>$T_s$</td>
<td>1.30</td>
<td>-0.13</td>
</tr>
<tr>
<td>$T_b$</td>
<td>0.40</td>
<td>1.28</td>
</tr>
<tr>
<td>$T_{mat}$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$T_{crit1}$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$T_{rew}$</td>
<td>-0.84</td>
<td>0.00</td>
</tr>
<tr>
<td>$dens$</td>
<td>-0.46</td>
<td>0.33</td>
</tr>
<tr>
<td>$K_{c,max}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_w$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\beta$</td>
<td>7.23</td>
<td>-0.70</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.95</td>
<td>-0.13</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.64</td>
<td>-0.13</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$z_{r,max}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$z_{r,init}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$t(z_{r,max})$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Root activity (Novak, 1987)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 11  LAI-SIM input parameter in decreasing order of sensitivity index SI

<table>
<thead>
<tr>
<th>Lavalette - run</th>
<th>Input parameter in the order of decreasing SI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Optimum conditions</strong></td>
<td>$-\beta + T_f - T_f - T_s + T_b - \delta_1 - LAI_{max} + LAI_{max} - T_{crit2} + \beta$ $-\delta_2 - dens - T_b + dens + \delta_1 + \delta_2 + T_s$</td>
</tr>
<tr>
<td><strong>Water stress conditions</strong> (CMS-o)</td>
<td>$-\beta + T_b + T_f - K_{c,max} - T_f - T_s - \lambda - \delta_2 - T_{crit2} - z_{r,max}$ $-LAI_{max} + t(z_{r,max})$ [Root distribution] $+\delta_2 - \alpha_w + z_{r,init}$ $-T_b + \lambda + K_{c,max} - z_{r,init} + LAI_{max} - \delta_1 - t(z_{r,max}) - dens$ $+\alpha_w - T_{mat} + TP(h_{d(1)}) - h_{d(2)} + z_{r,max} - T_s + dens + h_{critA}$ $-h_{critA} - h_{up} + h_{up} - h_{d(2)} - TP(h_{d(1)}) + TP(h_{d(2)}) + TP(h_{d(1)})$ $-TP(h_{d(2)}) + TP(h_{d(2)})$</td>
</tr>
</tbody>
</table>
In the case of the CMS-s run, the shape parameter $\beta$ also has the highest sensitivity on predicted LAI (SI = 8.14%). The highly sensitive thermal-time parameters of the CMS-o run ($T_r$, $T_s$, $T_b$) and $LAI_{\text{max}}$ are also extremely sensitive for the CMS-s run as seen in Table 10. The SI values for $T_r$ (+4.4%) and $T_b$ (+6.5%) are much higher under stressed conditions than as for CMS-o. Ranking fourth among the most sensitive parameters is the crop coefficient $K_{c,\text{max}}$ with a SI of +3.4%. A decrease of the $K_{c,\text{max}}$ value by one percent for the given stressed condition (CMS-s) results in an average increase of the LAI values by 3.4%. This is due to the decrease of the potential transpiration rate (Equation 104), the increase of the $\text{TA}/\text{TP}$ ration and, consequently, the increase of the stress index (i.e. the decrease of crop water stress). The parameter $\lambda$, related to the crop sensitivity to water stress, is moderately sensitive (SI = +0.85%). Similar in value (+0.7%) is the SI of the shape parameter $\delta_2$. All other parameters in the investigation are moderately or less sensitive to the LAI prediction. Similarly, the root growth is less sensitive with a maximum (absolute) SI value of 0.5% for the maximum root depth $z_{r,\text{max}}$. However, the root development is well known for most agricultural crops and under different soil conditions.

In the presented analysis, the root activity, i.e. water uptake intensity, is taken to be either equally distributed within the root zone or to increase exponentially with depth as explained in subsection 3.3.6.1. Using the nonlinear root activity model leads to an average LAI increase of only 0.5% (cf. Table 10) in the case of CMS-s. Nevertheless, it might have a stronger impact under different soil conditions.

The plant coefficient $\alpha_w$ has been discussed in subsection 3.3.1.4. Its value can vary for different crops from 0.6 to 0.82. The impact on LAI, however, is relatively low which is confirmed by a SI value of 0.4% only. As a result, no significant loss in accuracy of the predicted LAI is expected by using $\alpha_w = 0.7$ as proposed by Mailhol et al. [1997].

### 3.3.7 Complete furrow irrigation model FIM

An irrigated field is a dynamic system with relatively well-defined boundaries. This system consists of the three components: soil surface, the soil body and the crop. The presented furrow irrigation model FIM simulates the main water transport processes to, from and within this system. These are surface water flow during irrigation, infiltration of surface water and water transport within the soil matrix, water extraction by the plant rooting system, percolation, evaporation from the soil surface and penetration of water at the soil surface due to precipitation. Furthermore, crop development is simulated by means of leaf-area index, which depends on climatic conditions/input and on a water-stress factor calculated from actual soil moisture conditions.

In the preceding sections, mathematical models for the surface flow, the subsurface flow and crop growth (and crop water requirements) are developed and tested in detail. Coupled models of surface-subsurface flow as well as of the crop-subsurface interactions are derived and validated in subsections 3.3.5 and 3.3.6, respectively. These modules are now integrated into the furrow irrigation model system FIM. A comprehensive time management unit controls the transient boundary conditions of the irrigation system as well as the changing type of the boundaries due to the different ‘events’ (e.g. irrigation events or rainfall events) during the simulation of an irrigation season. It also governs the central time line in which the different sub-modules are synchronized under consideration of their individual time-step management.
Physically based modelling of the comprehensive furrow irrigation system is accompanied by a large number of input parameters which have to be derived carefully. Initial and boundary conditions also have to be evaluated from measurements or estimations. It goes without saying that model parameterization is rather complex and requires a well-structured tool. This is a user-friendly graphical interface (GUI) which was developed to make parameterization much easier. Moreover, the GUI presents the model results and thus enables the user to study the impact of design and management variables on crop development and yield.

A rather simple optimization technique is incorporated in the FIM for determining irrigation schedules, which guarantee close to optimum crop water supply for given parameterizations.

3.3.7.1 Modules

The main modules of the model system FIM are the surface flow model FAPS, the subsurface flow model HYDRUS-2D, the crop model LAI-SIM, the time management and control unit, as well as input and output modules. The principles and interaction between surface, subsurface and crop model have been outlined in detail in the previous sections. An integrated scheme of the interacting models in the FIM framework is displayed in Figure 39.

The model system is coded in a framework of program routines and functions integrated in the Matlab environment. One exception is the HYDRUS-2D code, written in Fortran90 for Windows. The original code is transformed to run under Linux/Unix operating systems and upgraded to Fortran95. An interface routine is programmed which transfers a complete

![Figure 39 Modules of the furrow irrigation model system FIM](image-url)
structure of HYDRUS-2D variables from the *Matlab* environment to the compiled *Fortran* code of HYDRUS-2D and back.

A time management and control routine organizes and synchronizes the events during the simulation as well as the data and variable exchange during the simulation as described later in this section. A time line controls the sequence, the time and duration of the events. At the beginning of the simulation, the model parameters/variables of HYDRUS-2D and LAI-SIM are initialized/allocated by initialization routines. In contrast, the surface model parameters are allocated at the precise moment when an irrigation event starts. Output variables are collected and saved both at defined standard times (e.g. before and after an irrigation event, at the end of each simulation day, etc.) and at user-defined times. At the end of the simulation, post-processing routines (Figure 39) calculate the final crop yield, the components of both the cross sectional and the total field water balance, and a number of performance criteria which are used to evaluate the simulation results.

### 3.3.7.2 FIM – model range of applicability

The range of applicability of the model system FIM is governed by the integration of the prescribed surface, subsurface and crop modules, and the individual assumptions leading to these modules. These assumptions are given in detail in the sections 3.3.2, 3.3.3, 3.3.4, 3.3.5 and 3.3.6.

FIM is designed to simulate irrigation events in arbitrarily shaped prismatic channels with a free draining lower boundary and a constant inflow rate at the upper boundary. Both the soil hydraulic parameters and the initial (soil moisture) conditions are either homogeneous or arbitrarily distributed in the channel cross section (soil layers). Irrigation management and design parameters (inflow rate, irrigation duration, furrow slope, surface roughness, etc.) can vary for subsequent irrigation events. An irrigation event must take place during a single day. Multiple, sequential irrigation events on a single day, however, can be simulated by FIM.

Subsurface water flow is simulated quasi three-dimensionally (3D) by utilizing HYDRUS-2D at predetermined (arbitrarily spaced) locations along the furrow. FIM simulates crop growth at these predetermined computational knots throughout the whole growing period. The FIM variables ($I$, $q$, $TA$, LAI, etc.) are assumed to change linearly between these predefined locations.

The daily averages of the climatic boundary conditions (e.g. precipitation) are uniformly applied at the soil surface. The plant density is assumed to be homogeneous along the furrow length and width. FIM is generally applicable to any crop. Validated crop model parameters are given for corn, sunflower, sorghum, tomato, potato, soybean and wheat (cf. Table 6).

### 3.3.7.3 Time management and event control

A comprehensive time management unit controls and synchronizes the events and the data transfer during the simulation. Events are the times when irrigation starts, the times when irrigation ends/redistribution starts, the times when the values and/or the type of the flow domain boundary conditions change, and the times of input and output. These times are organized in a time line of events.

Not all event times are known at the start of the simulation. For example, following an irrigation, the moment when the redistribution phase begins is determined by the duration of the surface-flow recession phase. Although cut-off time is predetermined for each irrigation,
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the total recession time, i.e. the time when all water has left the furrow, is a result of the model calculations. Thus, the time line is completed and/or modified in the course of the simulation.

Figure 40 shows a schematic plot of the event-based program flow of the model system FIM using chart symbols common in informatics. The boxes on the top line in Figure 40 stand for 'objects', which are software routines/functions grouped into generic terms. These objects are:

i) main program
ii) event control and time management unit
iii) crop model LAI-SIM
iv) sequence of HYDRUS-2D subsurface models
v) post-processing routines
vi) surface flow model FAPS divided into
   a) FAPS model frame and hydraulic phase control
   b) solution algorithm for the nonlinear set of differential equations.

The number of the HYDRUS-2D subsurface models – more precisely, the number of the individual HYDRUS-2D parameterizations – equals the number of predetermined calculation locations along the furrow. The right-hand side of Figure 40 gives the action list of subsequent events. Vertical lines indicate the time line which runs from the top to the bottom of the figure. Boxes along the time line indicate the domain (times) of action of the corresponding objects; full-line arrows are function calls including data transfer and broken-line arrows are the function response and/or data transfer to the superior object. Three dots in the plot indicate repeated actions, e.g. HYDRUS-2D calculations for \( i = [1, 2 .. n] \) parameterizations, or repeated iteration steps within an advance phase iteration loop.

Figure 40 shows three simulation days out of a total irrigation cycle: one day with irrigation and two days without irrigation. FIM always starts with data input and the initialization of the crop model and the allocation of the HYDRUS-2D data structures. The crop model initially calculates the potential evapotranspiration ETP from climatic data and the resulting potential LAI (i.e. the LAI function depends only on climatic data and not on soil water stress).

According to the exemplarily chosen irrigation schedule in Figure 40, the calculation starts on a day without irrigation. First, the boundary conditions of the HYDRUS-2D models (TP, EP, P, root depth and activity) are calculated by the crop model LAI-SIM and allocated in the corresponding data structures. Next, the event control unit determines the target time, initial time step and time control variables of the HYDRUS-2D model. Now HYDRUS-2D computes for all cross sections along the furrow within the time interval \([\text{das}, \text{das} + 1]\) by its own time-step management. Variables such as cross-sectional infiltration, actual evaporation and transpiration are cumulated within the given time interval\(^\text{16}\). At target time \([\text{das} + 1]\), the cumulative (cross-sectional) variables are transferred back to the main program and buffered for further calculations and post-processing. The final steps are repeated for the number of simulation days without irrigation, which is indicated by the three horizontal dots in the event-control time line of Figure 40.

\(^{16}\) The time interval is one day for simulation days without irrigation.
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Figure 40  Simplified time line of the model system FIM (chart symbols and description explained in section 3.3.7.3)
The simulation of an irrigation day starts in a similar fashion, but is divided into three sub-intervals:

1. \([\text{das}, t_\alpha]\]
2. \([t_\alpha, t_\omega]\]
3. \([t_\omega, \text{das} + 1]\)

where \(t_\alpha\) and \(t_\omega\) denote the beginning and end of the irrigation, respectively. Thus, sub-intervals 1) and 3) denote redistribution times, whereas sub-interval 2) is the total irrigation time.

The sub-interval 1) is treated similarly as for a redistribution day but with a target time \(t = t_\alpha\). At the beginning of the sub-interval 2), the FAPS model is initialized according to the event-specific parameters. The event control unit synchronizes the global simulation time (which holds true for the main program and for the HYDRUS-2D models) and the FAPS model time (which always starts with zero at the beginning of an irrigation event). This time shift/offset is also considered at each HYDRUS-2D function call from FAPS.

FAPS runs through the four phases of irrigation (Figure 40). As far as the times which mark the four phases are concerned, only the times when the water application begins and ends (i.e. \(t_\alpha\) and cutoff at time \(t_\omega\)) are known beforehand. The advance time \(t_\alpha\), depletion time \(t_r\) and total recession time \(t_\omega\) are results of the simulation. Thus, even the duration of the sub-intervals 2) and 3) is not known beforehand.

At the beginning of the iteration for the initialization of HYDRUS-2D, the iterative calculation of the advance and recession times requires a relaxation of the HYDRUS-2D parameterizations with the 'old' former values. Of course, all the atmospheric boundary conditions and the root water uptake likewise hold true during the sub-interval 2). The type and value of the surface boundary nodes of the HYDRUS-2D models change from prescribed flux to head boundary type (and vice versa) according to the transient flow depth which is calculated by FAPS (for details refer to section 3.3.5). At the end of irrigation, the calculated total recession time \(t_\omega\) is transferred to the event control unit, among other variables, and stored in the time-line of events. Now, the redistribution phase starts again in the sub-interval 3) as described above.

The sequence of days with or without irrigation continues according to the irrigation schedule. Finally, the simulation stops at the day of harvest. Comprehensive post-processing program routines calculate both the cross-sectional and the total volume balance components, the irrigation performance indices and the final crop yield.

The tasks of the time management and event control unit are summarized as:

i) the controlling and synchronization of the simulation time line
ii) the time step control of the different modules in the FIM model system
iii) the data transfer including data input and output
iv) the allocation and initialization of HYDRUS-2D parameter structures
v) the controlling of HYDRUS-2D boundary condition values and types.

Bearing its important functions in mind, the control unit is to be considered as the 'heart' of the operation of the interlinked models in the model system FIM.
3.3.7.4 Graphical user interface

Due to the physical basis and complexity of the model system, a large number of input data is required to operate FIM. Therefore, a graphical user interface (GUI) was developed which supports and guides the user during the model parameterization and displays the simulation results for further analysis. Input data can be entered and modified easily by the GUI. Moreover, parameters are grouped into parameterization blocks (e.g. crop model parameter block, surface geometry parameter block) which can be saved and then later reloaded and modified. A scenario management is also included in the GUI, which allows saving the input and output data of the entire simulation runs. This feature enables the GUI to support various studies. At one and the same time it allows for the analysis of simulation results which are not only presented graphically but are also exported for further processing. The GUI consists of a main scenario (control) window and twelve input and output windows, which are subsequently presented.

Scenario window

The GUI starts with the scenario window (Figure 41). Here, the irrigation scenario can be composed by choosing predefined parameter blocks. These three parameter blocks are indicated in Figure 41: (1) the furrow geometry block, (2) the crop data block and (3) the root growth and stress-response parameter block. These parameter blocks can be created, modified and saved in three other GUI windows, namely, the geometry input window, the crop data window and the root-growth/water-stress data window.

As seen in Figure 41, the irrigation schedule, i.e. the irrigation date, the time, the duration and the corresponding inflow rate, can be entered into the schedule box (5). Further, the dates of desired pressure-head distribution output can be defined by the user (4). Once the simulation
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has taken place on these dates, the calculated pressure-head distribution can be visualized in the pressure-head window.

Some FIM control parameters are assigned with default values: the number of infiltration locations, the time step for the storage phase (S-II), the time step for the depletion/recession phase and the tolerance criteria for subsequent FAPS iterations. In the GUI, the number of infiltration calculation locations is by default five, i.e. the field divided into quarters. With five equidistant cross sections it is possible to calculate commonly used irrigation performance criteria. Optionally, two locations only (the inflow and outflow section) can be chosen for rough but fast calculations.

Symmetrical furrow (7) is an option, which significantly reduces computation time. By choosing it, the user parameterizes only half of the subsurface flow domain. The water flow is assumed to be symmetrical at both sides of the furrow's vertical centre line.

A help box (6) exists throughout all GUI-windows and provides a brief description of the window contents, the input/output parameters and special hints for the user. A navigator tree (9) at the right hand side of the windows allows rapid alternation between the GUI windows.

A menu bar ('scenario') makes it possible to create, open, save, delete and import entire simulation scenarios. This feature also allows for the visualization of previously calculated scenarios without having to run them through again. The simulation of the given scenario is also initiated from this menu bar.

Automatic irrigation control
In the scenario window (Figure 41), the user can choose between Manual Schedule and Automatic Irrigation Control (8). With the first choice, the irrigation times, the duration and the inflow rate are user-defined. With the second choice, FIM calculates the irrigation date and time (i.e. the schedule) automatically during the course of the simulation in the following manner.

At the beginning of each simulation day, the average soil moisture content in the root zone is calculated and this directly yields the soil moisture deficit. This deficit is compared to a threshold value which is defined by the user (Figure 41). If the soil moisture content falls below the critical value, the field is irrigated with the user-defined inflow rate. The cut-off time is determined by controlling (calculating) the root-zone water content at each time interval of the simulation and by comparing it with a second (upper) threshold value. If this target soil moisture content is reached, the water application (inflow) is cut off.

The lower and upper critical soil moisture deficit is set to 30% and 70%, respectively, and the inflow rate is set to 0.0005 m³/s. The actual resulting crop almost yields its full potential value, but runoff is comparably high with 43% of the total irrigation water.

Automatic irrigation control in FIM corresponds to the farmer's practice of irrigation on demand. The demand can only be roughly determined in the field by local measurements of either soil moisture content (e.g. TDR probes) or pressure head (e.g. tensiometer probes) or, sometimes the preferred method, simply by the farmer's experience.

Furrow geometry input
Furrow geometry parameters are entered into the geometry input window (Figure 42) and can be saved as a parameter block. The furrow cross section is described by the general hydraulic functions Equations 69 and 70 as shown in subsection 3.3.2.1.
Often, the hydraulic parameters are not known beforehand but measurements of the furrow cross section are accessible or can be conducted with comparably little effort (cf. Wöhling [1999] for the methodology). In order to facilitate the parameterization of the hydraulic section, a data-fit routine is developed for calculating the parameter of the general type cross section $p_1$ to $p_4$ from measurements of the $y - z$ coordinates of the furrow cross section. Alternatively, the parameters of a triangular cross section can be chosen and the parameters of the general type cross section are derived mathematically by FIM.

**Crop parameter input**

The parameterization of the crop model is conducted in the crop parameter window (Figure 43). It includes the input of both parameters of the LAI-SIM module and yields model parameters. Just to the left of the navigator tree (Figure 43) the parameterization of the selected crops can be chosen from the database, which originates from Mailhol et al. [1997] and Mailhol [2003]. A list of these crops and the corresponding parameters is given in Table 6. The user can enter/modify the parameters himself and thus create new parameter blocks which are then saved in the crop data base.

**Root growth and water stress parameters**

Both the root growth parameters and the parameters of the water stress response function by Feddes et al. [1978] are entered into the root growth and water stress data window (Figure 44). These parameters must correspond to the crop which is chosen in the crop parameter window (Figure 43).

The root growth and water stress parameters are explained in detail in subsections 3.3.6.1, 3.3.6.1 and 3.3.6.2. In the input window, the stress response function is visualized for transpiration rates corresponding to the critical lower/upper turgor pressure (cf. subsection 3.3.6.2). As with the procedure for the crop, parameters can be chosen from the database. The parameters can also be entered, modified and saved in the database by the user.
Subsurface flow parameterization
The software package HYDRUS-2D is used for the generation of the subsurface flow domain, the calculation mesh, the parameterization of the initial and boundary conditions, and the parameterization of the soil hydraulic model. HYDRUS-2D default values can be used for the root water uptake parameters, the time discretization, the iteration criteria and the output options because all these parameters are externally controlled by FIM. Time-variable atmospheric boundary conditions are also not specified, since these are likewise controlled by
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FIM. HYDRUS-2D creates the input files meshtria.txt, domain.dat, boundary.in, dimensio.in and selector.in. The name of these input files and the directory path is entered/chosen in the FIM-GUI subsurface flow window (Figure 45). A preview of the file contents is also available.

Climate data
A text file with climate data records for the growing season is required. This must contain the daily values of reference evapotranspiration \( ET_0 \ [\text{mm d}^{-1}] \), the mean air temperature \( T[^\circ C] \), precipitation \( P[\text{mm d}^{-1}] \) and solar radiation \( SR [\text{Jm}^{-2}] \). The daily records are consecutively numbered in the first column of the file. The total number of climate records, i.e. the number of days, is used by FIM to determine the total simulation time. The first record in the climate file is associated with the day of sowing, whereas the last entry is associated with the day of harvest.

The climate data file name and directory path in the given file system must be entered/chosen in the FIM-GUI climate data window (Figure 46). A preview of the file contents is also shown in this window.

General output data
Various FIM output data are displayed in the output window (Figure 47):

- the simulation time, the CPU time requirement
- the components of the total flow-domain water balance, i.e. the volume of irrigation water (inflow), the infiltration volume, the runoff volume, the volume of transpiration, evaporation and percolation
- the relative volume balance error
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- the irrigation performance criteria of the single irrigations (IE, AE, DU, AD\textsubscript{a}), the overall irrigation efficiency
- the potential and actual crop yield.

Figure 46 FIM-GUI – climate data window

Figure 47 FIM-GUI – general output window
The yield is calculated by (i) the methodology of Mailhol et al. [2004] and (ii) by the FAO33 standard methodology [Doorenbos and Kassam, 1986]. It is related to the entire field by multiplying the yield estimated for the single furrow by the field's total number of furrows.

The time and date of simulation start/end is also shown in the output window for an exact identification of the scenarios.

**Irrigation advance and recession times**
Irrigation advance and recession times are plotted over the furrow length in the FIM-GUI surface flow window (Figure 48) in the left plot box and right plot box, respectively. The data corresponds to the irrigation event which is chosen by the pop-up menu (Figure 48).

![Figure 48 FIM-GUI – surface flow window](image)

**Plant water and atmospheric data**
The simulated daily values of both processes relating to plant water and climatic data are plotted over time in the FIM-GUI water fluxes window (Figure 49) for each HYDRUS-2D calculation cross section. By activating the respective check box, the following functions are plotted:
- the potential and actual evaporation
- the potential and actual transpiration
- the precipitation
- the maximum potential transpiration (i.e. the crop being without water stress during the entire growing season).

The numerical data of these functions are visible by choosing the corresponding sub-folder in the navigator tree.
Predicted LAI
The potential leaf-area index and the simulated actual leaf-area index are shown in the LAI window (Figure 50) for each HYDRUS-2D calculation cross section. It assists the user in evaluating possible differences in crop development at the predefined computational knots along the furrow length. Potential leaf area is calculated by assuming optimum crop-growth conditions, i.e. no water stress.
Water content
The total field water content of the subsurface flow domain is calculated by (linearly) integrating the calculated water content at the HYDRUS-2D computational knots along the furrow. It is plotted as a function of time in the left-hand plot box of the FIM-GUI water content window (Figure 51). It should be noted that this is the water content of the total flow domain and not the water content of the root zone. The peaks in this plot (Figure 51, left plot box) clearly indicate the irrigation events.

![Figure 51 FIM-GUI – water content window](image)

The right-hand plot box of the FIM-GUI water content window (Figure 51) shows the water content at the HYDRUS-2D computational knots over the furrow length at times before and after the irrigation events, respectively. This plot enables the user to analyze the effect of the individual irrigation events. It can also be used for the analysis of the application uniformity.

Pressure head distribution
The pressure head distribution at the predetermined HYDRUS-2D calculation cross sections is shown in the FIM-GUI pressure head window (Figure 52). Two plot boxes allow the comparison of the pressure head distribution of (i) two different locations at a selected simulation time, or of (ii) a single location at two different times (e.g. before and after irrigation). The rooting depth is indicated by a white horizontal line.

Preview boxes on the left side of the two plot boxes show the entire subsurface flow domain, whereas the plot boxes themselves may show only the upper part of it. The user can navigate up and down the flow domain with the sliders on the left of both plot boxes. Synchronous navigation of the two plot boxes is possible if the respective check box is ticked.

List boxes enable the user to choose between the locations of the cross section as well as between the output dates. Fixed output dates are the days both before and after the irrigation events as well as the day of sowing (initial pressure head distribution) and the day of harvest (final pressure head distribution). Supplementary output dates are defined by the user prior to the simulation in the FIM-GUI scenario window (cf. paragraph 3.3.7.4).
Inflow/infiltration ratio

The water application volume (inflow) of the individual irrigation events is plotted throughout the duration of the growing period in the FIM-GUI inflow/infiltration window (Figure 53). It also shows the proportion of the inflow volume which has entered the soil by infiltration. The inflow/infiltration window displays the irrigation schedule, i.e. the time intervals between successive irrigation events and gives an impression of the percentage of the irrigation water which is lost by surface runoff.
Methods: the modules of GAIN-P

3.4 Global optimization of irrigation control and scheduling: the interaction of the GAIN-P sub-modules

The interaction between the sub-modules of the GAIN-P methodology, namely, the furrow irrigation model FIM, the new SOM-MIO neural network and the problem-adapted evolutionary algorithm, opens up new horizons as regards the simultaneous optimization of irrigation control and scheduling.

In the past, the problem of irrigation scheduling and the determining of the control parameters in furrow irrigation were considered as two separate items. The first problem of irrigation scheduling under limited seasonal water supply has been studied extensively [Bras and Cordova, 1981; Rao and Sarma, 1990; Sunantar and Ramirez, 1997]. In these studies optimization techniques such as dynamic programming (DP) have been used to determine the optimal operation policies in irrigation systems. It is an unfortunate fact, however, that the computational requirements of DP become overwhelming when the number of state and control variables is very large [Bellmann and Dreyfus, 1962]. For this reason, several restrictions such as fixed application intervals as well as the usage of simplified water balance models are incorporated in the DP-based optimization tools. With their economic optimization technique, Raghuwanshi and Wallender [1997b] attempted to use more sophisticated models. They constructed a seasonal furrow irrigation model based on kinematic-wave hydraulics to minimize seasonal irrigation cost for a prescribed irrigation adequacy. This technique, however, likewise comprises some restrictions due to (i) employment of the empirical, though spatially variable, Kostiakov infiltration, (ii) a constant irrigation interval and (iii) ‘optimization’ with systematic simulation rather than with a programmed nonlinear optimization technique. The new GAIN-P strategy overcomes these restrictions by combining rigorous process-based furrow irrigation modelling with a new ANN technique and a tailor-made evolutionary optimization strategy, which accounts for both variable irrigation intervals and variable irrigation parameters.

Optimizing both control and schedule parameters in furrow irrigation is considered a nested problem (Figure 54): (i) optimizing control parameters for each single water application, which is referred to as ‘inner optimization’ and (ii) optimizing the irrigation schedule (i.e. number and date of applications) over the whole growing season, which is referred to as ‘outer optimization’. The objective of the global (nested) optimization is to achieve maximum crop yield with a given, but limited, water volume, which can be arbitrarily distributed over an adequate number of irrigations. The impact of different irrigation schedules on crop yield is calculated by the furrow irrigation model (Figure 94).

It is difficult to solve the global optimization problem because the target function has many locally optimal solutions and features an undefined number of optimization variables because the number of irrigations is a priori unknown. Thus, finding the global solution is not possible with classical deterministic optimization techniques. For this reason, a made-to-measure evolutionary optimization technique (EA) is employed to find a near-optimal solution of the outer optimization problem (when and how much to irrigate) within an acceptable computation time.

For efficiently solving the inner optimization problem, namely the determination of the control parameters for each water application (inflow and cut-off time), a problem-adapted artificial neural network (ANN) based on self-organizing maps (SOM) was developed (see section 3.1). The new architecture allows simulation tasks to be performed as well as inverse problems to be solved after a single training: the self-organizing map with multiple input/output option (SOM-MIO). The SOM-MIO portrays the inverse solution of the coupled numerical
surface/subsurface flow model and thus enormously speeds up the overall performance of the complete optimization tool.

For training the SOM-MIO with realistic scenarios we apply the rigorous and physically sound coupled surface-subsurface flow model (see section 3.3). The model is based on an analytical zero-inertia surface flow model iteratively coupled with the numerical code HYDRUS-2D, which simulates subsurface flow by the modified Richards equation. In addition to the obtained accuracy, this also permits consideration of soil evaporation and precipitation as well as root water uptake by plants.

Evolutionary algorithms (see section 3.2) represent an alternative to classical optimization methods when dealing with objective functions which feature many local minima. They imitate genetic principles from nature. A GA begins its search with a random set of solutions called 'population' which, in our case, is a random set of schedules. Every solution is assigned a fitness value which is directly related to the objective function of the optimization problem, i.e. the estimated crop yield. Thereafter, the population of schedules is modified to a new population by applying four steps similar to natural genetic operators – selection, crossover, mutation and reconstruction.
4 Case studies: application of GAIN-P strategy

Before dealing with the optimization problem, the physically based furrow irrigation model FIM is applied to simulate laboratory and field experiments which consider a whole growing season. This analysis serves to validate the overall qualities of FIM with respect to reliably predicting the results of different water application strategies.

4.1 Simulation of furrow irrigation in practice

4.1.1 Experimental data

A number of frequently used data on furrow irrigation advance (and recession) experiments can be found in technical literature where infiltration characteristics are expressed in terms of the empirical modified Kostiakov equation (38). However, no publication is known which provides (i) observations of the temporally and spatially varying, quasi 3D water flow in irrigated furrows and (ii) a full set of parameters for the physically based modelling of the flow processes including, for example, the parameters of the 2D Richards equation.

For this reason, 'close-to-nature' experiments were conducted over a period of two years at the Hubert-Engels Laboratory at the Institute of Hydraulic Engineering and Applied Hydromechanics, Dresden University of Technology (TUD).

Field experiments for the evaluation of crop growth (corn) were conducted at CEMAGREF in Montpellier, France. These closed-end furrow (CEF) experiments did not include measurements of the (2D) soil moisture distribution. With some reservations, however, they are suitable for the validation of the new furrow irrigation model.

More field experiments have been successfully conducted at the Indian Institute of Technology (IIT), Kharagpur, India as part of another cooperative programme accompanying this contribution.

The experimental data from literary sources, CEMAGREF and the IIT Kharagpur are subsequently presented in a rather condensed form. For the case that specific data has already been published, the literature references are given for more detailed reading in the respective sections. Because of the uniqueness of the experiments at the Hubert-Engels Laboratory, their setup, conduction and results are discussed in more detail in this section.
4.1.1.1 Data from literature

Walker and Humphreys [1983] presented field experimental irrigation data of continuous flow from three US sites in Colorado, Idaho and Utah. In all the experiments, infiltration is described by the Kostiakov equation. Elliott et al. [1982] used a number of runs from the Colorado data (e.g. the irrigation experiments 'Printz 3-2-3' and 'Matchett 2-3-5') which represent a wide range of field conditions. The Colorado data, however, is not completely independent because the parameters of the Kostiakov equation ($k_0$ and $a_0$) were computed from advance data [Walker and Humphreys, 1983]. On the other hand, the data of the experiments, which are referred to as 'Flowell' and 'Kimberly', are representative data obtained from a series of tests conducted in Utah and Idaho. All the parameter values of these data sets are based on independent field measurements [Mostafazadehfard, 1982; Malano, 1982].

Another source of data is the large number of furrow irrigation tests conducted at three field sites in south-east Australia. Esfandiari [1997] used these data (Kostiakov infiltration) for an extensive comparison of different furrow irrigation models. Among others, the experiments FS47 and WP11 were conducted at the Field Services Unit Paddock farm of the University of Western Sydney-Hawkesbury at Richmond, 60 km west of Sydney and at Walla Park, Quirindi about 350 km north of Sydney, respectively.

Using the Kostiakov infiltration approach, selected experimental data from the above mentioned sources are used for the verification of the coupled surface-subsurface model and are given in Table 12.

4.1.1.2 Laboratory experiments

Various furrow irrigation experiments were conducted at, and in cooperation with, the Institute of Hydraulic Engineering and Applied Hydromechanics, Dresden University of Technology in 1999 and 2000 [Dresden University of Technology research report, 2000]. In the course of the presented study, the experimental setup and operation was designed so that the interacting surface and subsurface water flow phenomena could be observed in high resolution in space and time during furrow irrigation. A similarly comprehensive experimental setup at laboratory scale (but under 'near field conditions') is not known from literature. Bridging the gap between laboratory scale and field experiments is, however, essential in order to test and validate the coupled flow model. Two series of six subsequent irrigation experiments were conducted during a period of two years with a six-month break for replacing the silty loam soil used in the first experimental series with a sandy silt soil. Selected experiments with the silty loam soil are used in this study.

Laboratory setup

At the Hubert-Engels Laboratory, a 26.4 m long, 0.88 m wide and 1.0 m deep experimental tank was constructed using steel profiles combined with Plexiglas. For the first series of experiments (Figure 55), the tank was carefully filled layer by layer and compacted uniformly with about 50 tons of silty loam soil to achieve, as far as possible, a homogeneous distribution of the soil properties (saturated moisture content = 41%, residual moisture content = 14%, bulk density = 1.54 Mg/m³, particle density = 2.62 Mg/m³). A parabolic furrow, with a top width of 0.35 m and an initial depth of 0.184 m, was formed along the central longitudinal axis of the tank. The furrow shape function is defined by: $h_{w}(y_f) = 5.86y_f^{1.09} + 0.18$, where $y_f$ is the furrow width, with the origin at the vertical furrow central line (Figure 56).
Table 12  Furrow modelling input data for the coupled surface-subsurface flow model

<table>
<thead>
<tr>
<th></th>
<th>Matchett 2-3-5</th>
<th>Printz 3-2-3</th>
<th>Flowell-wheel</th>
<th>FS47</th>
<th>WP11</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Soil type</strong></td>
<td>Clay loam</td>
<td>Loamy sand</td>
<td>Sandy loam</td>
<td>Sand / Sandy loam</td>
<td>Clay (Vertisols)</td>
</tr>
<tr>
<td><strong>Field length</strong> $x_L [m]$</td>
<td>425</td>
<td>350</td>
<td>360</td>
<td>59</td>
<td>941</td>
</tr>
<tr>
<td><strong>Slope</strong> $S_0 [m m^{-1}]$</td>
<td>0.0095</td>
<td>0.0025</td>
<td>0.008</td>
<td>0.0032</td>
<td>0.0014</td>
</tr>
<tr>
<td><strong>Mannings</strong> $n$</td>
<td>0.02</td>
<td>0.02</td>
<td>0.04</td>
<td>0.028</td>
<td>0.017</td>
</tr>
<tr>
<td><strong>Inflow</strong> $Q_{in} [m s^{-1}]$</td>
<td>0.00092</td>
<td>0.00349</td>
<td>0.002</td>
<td>0.00175</td>
<td>0.00208</td>
</tr>
<tr>
<td><strong>Kostiakov- $k_k [m^3 m^{-1} s^{-0.5}]$</strong></td>
<td>6.416 $\cdot \ E^{-4}$</td>
<td>0.01132 $\cdot \ E^{-4}$</td>
<td>3.145 $\cdot \ E^{-4}$</td>
<td>2.56 $\cdot \ E^{-4}$</td>
<td>0.0227</td>
</tr>
<tr>
<td><strong>Kostiakov- $a_k$</strong></td>
<td>0.4</td>
<td>0.024</td>
<td>0.534</td>
<td>0.525</td>
<td>0.1223</td>
</tr>
<tr>
<td><strong>Kostiakov- $c_k [m^3 m^{-1} s^{-1}]$</strong></td>
<td>$5.0 \cdot \ E^{-7}$</td>
<td>$8.18 \cdot \ E^{-6}$</td>
<td>$3.67 \cdot \ E^{-6}$</td>
<td>$8.33 \cdot \ E^{-6}$</td>
<td>$1.0 \cdot \ E^{-6}$</td>
</tr>
<tr>
<td><strong>Time of cut-off</strong> $t_{cc} [min]$</td>
<td>1364</td>
<td>110</td>
<td>400</td>
<td>53.5</td>
<td>962</td>
</tr>
<tr>
<td><strong>Hydraulic section parameter, $p_1$</strong></td>
<td>2.18</td>
<td>0.615</td>
<td>0.3269</td>
<td>2.03</td>
<td>1.48</td>
</tr>
<tr>
<td>$p_2$</td>
<td>0.79</td>
<td>0.70</td>
<td>0.536</td>
<td>0.80</td>
<td>0.81</td>
</tr>
<tr>
<td>$p_3$</td>
<td>1.25</td>
<td>0.695</td>
<td>0.4323</td>
<td>1.074</td>
<td>0.862</td>
</tr>
<tr>
<td>$p_4$</td>
<td>0.75</td>
<td>0.693</td>
<td>0.793</td>
<td>0.75</td>
<td>0.758</td>
</tr>
<tr>
<td>$a$</td>
<td>$2.48 \cdot \ E^{-3}$</td>
<td>$2.64 \cdot \ E^{-3}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$b$</td>
<td>2.176</td>
<td>1.923</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 55  Irrigation on *Lolium Multiflorum* at the experimental tank in the Hubert-Engels Laboratory, Dresden University of Technology (photo: Th. Wöhling)
A magnetic-inductive flow meter recorded the inflow to the furrow. A Laser Doppler Velocity meter (LDV) was used to measure the velocity of the surface flow at nine cross sections. In addition, ultrasonic probes were installed at these sections to measure the flow depth during irrigation. Altogether, 100 tensiometers and TDR\(^1\) probes were placed at five cross sections (\(x_{\text{inf}} = 1.5, 6.3, 12.3, 18.3\) and 24.3 m) to register matric head and soil moisture. The comprehensive instrumentation of the cross sections is portrayed in Figure 56. All measurements were continuously recorded by four data loggers. A special type of database was developed to store, access and organize the large amount of data which was accumulated during the long-term experiments.

Six irrigation experiments were conducted on the silty loam soil on 30 August, 18 October and 24 November 2000 as well as on 6 February, 28 February and 10 April 2001. The experiments differed from each other mainly in the water inflow rate and water application time. For three selected runs, the irrigation parameters, the hydraulic parameters and the geometrical characteristics are given in Table 13. The mean initial matric head for these runs is derived from tensiometer readings before the irrigation starts (Table 14).

\(^1\) TDR = Time Domain Reflectometry
Case studies: application of GAIN-P strategy

Table 13 Specifications of the laboratory experiments

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Run1</th>
<th>Run4</th>
<th>Run5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average inflow rate, $Q_{in}$ $[L\cdot s^{-1}]$</td>
<td>1.20</td>
<td>2.14</td>
<td>2.58</td>
</tr>
<tr>
<td>Cut-off time, $t_{co}$ $[hr]$</td>
<td>2.55</td>
<td>5.37</td>
<td>3.62</td>
</tr>
<tr>
<td>Furrow length, $x_L$ $[m]$</td>
<td>26.4</td>
<td>26.4</td>
<td>26.4</td>
</tr>
<tr>
<td>Furrow slope, $S_0$ $[m\cdot m^{-1}]$</td>
<td>0.0025</td>
<td>0.0014</td>
<td>0.0015</td>
</tr>
<tr>
<td>Roughness, $K_{st}$ $[m^{1/3}\cdot s^{-1}]$</td>
<td>33.3</td>
<td>33.3</td>
<td>33.3</td>
</tr>
<tr>
<td>Hydraulic section, $p_1$</td>
<td>1.494</td>
<td>1.909</td>
<td>1.752</td>
</tr>
<tr>
<td>Hydraulic section, $p_2$</td>
<td>0.665</td>
<td>0.789</td>
<td>0.775</td>
</tr>
<tr>
<td>Hydraulic section, $p_3$</td>
<td>0.449</td>
<td>0.530</td>
<td>0.539</td>
</tr>
<tr>
<td>Hydraulic section, $p_4$</td>
<td>0.540</td>
<td>0.590</td>
<td>0.598</td>
</tr>
</tbody>
</table>

Table 14 Laboratory experiments – initial matric head in the soil

<table>
<thead>
<tr>
<th>Soil depth $[m]$</th>
<th>0.20</th>
<th>0.30</th>
<th>0.35</th>
<th>0.45</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
<th>1.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_m$ $[m]$, Run1</td>
<td>-4.37</td>
<td>-3.33</td>
<td>-2.83</td>
<td>-2.41</td>
<td>-2.25</td>
<td>-2.02</td>
<td>-1.97</td>
<td>-2.07</td>
<td>-3.46</td>
</tr>
<tr>
<td>$h_m$ $[m]$, Run4</td>
<td>-6.65</td>
<td>-5.45</td>
<td>-6.02</td>
<td>-3.94</td>
<td>-4.09</td>
<td>-4.22</td>
<td>-3.61</td>
<td>-2.72</td>
<td>-3.43</td>
</tr>
<tr>
<td>$h_m$ $[m]$, Run5</td>
<td>-1.74</td>
<td>-1.45</td>
<td>-1.58</td>
<td>-1.16</td>
<td>-1.19</td>
<td>-1.06</td>
<td>-0.86</td>
<td>-0.64</td>
<td>-0.68</td>
</tr>
</tbody>
</table>

Water extraction from the soil was controlled by a grass cover along the central axis of the furrow (Figure 55). For this purpose, a perennial grass which is suitable for laboratory conditions (the ryegrass *Lolium multiflorum*) was chosen. Ryegrass was chosen not only because of its reported robustness under a wide range of environmental conditions, it also has a relatively high potential transpiration rate and a reasonably high rooting depth of more than 0.6 m (depending on soil conditions). The experimental tank was illuminated for 10 hours per day in order to ensure optimal plant growth (and, by this, maximum root water uptake). A lighting system with PHILIPS SON-T AGRO 400W light bulbs was used, which delivers a special radiation spectrum supporting vegetative growth.

Determination of soil hydraulic parameters

Soil hydraulic parameters were estimated by inverse solution using pressure head measurements at the cross section $x = 12.3$ m of the first experimental run. The objective function $Z$ is given by the mean square difference between measured and calculated soil water pressure:

$$Z(K_s, \alpha, n) = 0.5 \sum_{\text{probe}=1}^{14} \left( [h_{\text{meas}}(\tau, x) - h_{\text{sim}}(\tau, x)] \right)^2 \rightarrow \text{Minimum}$$

(166)

where $h_{\text{meas}}(\tau, x)$ denotes the measured pressure head and $h_{\text{sim}}(\tau, x)$ is the corresponding pressure head simulated by HYDRUS-2D. The saturated hydraulic conductivity $K_s$ and VGM model parameters $\alpha$ and $n$ represented the decision variables of the optimization problem. The values of $\theta_s = 0.41$ and $\theta_r = 0.14$ were determined from soil analysis and were taken as constants during the optimization runs. HYDRUS-2D includes a routine for inverse parameter estimation, which, however, often ends up in local minima of the objective function, especially if the parameter space of the decision variables is not adequately bounded. To overcome this well-known problem of the inverse method [Gribb, 1996; Schmitz *et al.*, 2004], a Matlab
routine was developed which systematically varies the three decision variables simultaneously in order to identify the global minimum of the objective function. Values of the error function are calculated by the mean square deviation of the simulated pressure head values from the tensiometer measurements. Figure 57 shows an example plot of the error function for the variation of the VGM parameters $\alpha$ and $n$.

\[
\text{Figure 57} \quad \text{Inverse soil hydraulic parameter estimation – error function for varying van Genuchten parameters } \alpha \text{ and } n
\]

The optimal values of the parameters $K_s$, $\alpha$, $n$ were evaluated according to Equation 166. Visual checks of the generated corresponding error maps (e.g. Figure 57) ensured that the result of the calculations did not end up in a local minimum. The final parameter set of the VGM model is obtained as: $\alpha = 1.4\text{m}^{-1}$, $n = 1.25$, $K_s = 1.95 \cdot 10^{-5}\text{m/s}$, $m = 1 - 1/n$ for $\theta_s = 0.41$, and $\theta_r = 0.14$. Since the irrigation experiments were conducted on homogeneous soil, these parameters were applied for the entire soil body.

4.1.1.3 Field experiments at Kharagpur, India

During 2003, irrigation experiments were conducted in eastern India as part of the cooperation between the Institute of Hydrology and Meteorology at the Dresden University of Technology and the Indian Institute of Technology (IIT) Kharagpur, India. Five irrigation experiments were conducted on three 40 m long furrows of sandy soil (Figure 58) with a slope of 0.005 m/m and an estimated roughness-coefficient of $K_{st} = 25 \text{m}^{1/3}\text{s}^{-1}$. The inter-furrow spacing was 0.75 m. The data presented in this section were collected at the central furrow.
The soil of the experimental plot has four layers as characterized in Table 15. Pressure head and soil moisture were measured at 0.5, 13, 26 and 39 m distances along the furrow at a depth of 0.15, 0.30, 0.45 and 0.60 m, respectively. The tensiometer probes were placed 0.1 and 0.15 m from the centre of the furrow ridge and connected to a data-logger for automatic recording every five minutes.

Table 15 Soil hydraulic characteristics of the experimental site in Kharagpur (India)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Layer [m]</th>
<th>Sand [%]</th>
<th>Silt [%]</th>
<th>Clay [%]</th>
<th>Bulk density [g·cm⁻³]</th>
<th>θₛ</th>
<th>θᵣ</th>
<th>α [m⁻¹]</th>
<th>n</th>
<th>Kₛ [m·s⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00 - 0.15</td>
<td>68.5</td>
<td>17</td>
<td>14.5</td>
<td>1.54</td>
<td>0.40</td>
<td>0.08</td>
<td>9.6</td>
<td>2.78</td>
<td>1.03·10⁻⁵</td>
</tr>
<tr>
<td></td>
<td>0.16 - 0.30</td>
<td>66.25</td>
<td>18.5</td>
<td>15.25</td>
<td>1.56</td>
<td>0.39</td>
<td>0.08</td>
<td>8.0</td>
<td>2.76</td>
<td>6.6·10⁻⁶</td>
</tr>
<tr>
<td></td>
<td>0.31 - 0.45</td>
<td>63.25</td>
<td>20.75</td>
<td>16</td>
<td>1.58</td>
<td>0.38</td>
<td>0.08</td>
<td>6.6</td>
<td>2.73</td>
<td>4.1·10⁻⁶</td>
</tr>
<tr>
<td></td>
<td>0.46 - 0.60</td>
<td>60</td>
<td>24</td>
<td>16</td>
<td>1.61</td>
<td>0.38</td>
<td>0.08</td>
<td>5.5</td>
<td>2.73</td>
<td>2.8·10⁻⁶</td>
</tr>
</tbody>
</table>

The five irrigations were carried out with different discharge and cut-off times. The first and the last ones, with the lowest and highest discharge respectively, were chosen for model testing. The parameterization of these two runs and the average initial matric head in the soil are given in Table 16 and Table 17.
4.1.1.4 Field experiments at Montpellier, France

Extensive field experiments were carried out over a 4-year period (1998-2001) on a loam soil plot (38% sand, 44% silt, 18% clay) under corn (variety Samasara) at the CEMAGREF experimental site Lavalette in Montpellier, France (Figure 59). Different irrigation and fertilizer treatments were applied in order to find the best irrigation and fertilizer strategy for the furrow (and sprinkler) irrigation technique. Besides the standard measurements pertaining...
to the surface flow, the experimental plot was also equipped with soil moisture and matric head sensors. Access tubes for measuring soil moisture were placed at the ridge and furrow bed of furrow number 60 at distances of \( x = 20 \text{ m}, \) \( x = 65 \text{ m} \) and \( x = 110 \text{ m} \). Additional measurements, e.g. LAI, provided a complete set of furrow irrigation data for a whole growing season [Mailhol, 2001; Mailhol et al., 2001; Nemeth, 2001]. Detailed and mostly unpublished data regarding volumetric soil moisture content, ETP, air temperature, precipitation and other aspects of the experiments were kindly provided by J.C. Mailhol from CEMAGREF.

During the experiments in 1999, around 30 blocked-end furrows with an inter-furrow spacing of 0.8 m were irrigated at the 130 m long Ta-plot of Lavalette. The longitudinal furrow slope was 0.25% and the roughness coefficient was estimated to be \( K_{st} = 20 \text{ m}^{1/3} \text{s}^{-1} \) for the first and \( K_{st} = 25 \text{ m}^{1/3} \text{s}^{-1} \) for the second and third irrigations. Furrow cross sections were almost trapezoidal with a bottom width of 0.1 m, a depth of 0.15 m and a top width of 0.4 m. Corresponding hydraulic section parameters are calculated to \( \rho_1 = 1.27, \rho_2 = 0.66, \rho_3 = 0.49 \) and \( \rho_4 = 0.57 \). The soil-profile of the Ta-plot is subdivided into three layers with the soil hydraulic characteristics being determined experimentally by direct and inverse methods [Mueller, 2001]. The parameters of the VGM model are given in Table 19. The parameter \( m \) of the VGM model is taken as \( m = 1 - 1/n \).

The corn was sown on 26 May 1999 and initially watered (28 mm) by a travelling gun system (sprinkler) in order to insure a homogeneous crop emergence. Due to over-average rainfall events (114 mm from June to the end of September), the plot was irrigated only three times: 76 mm on 10 July, 67 mm on 22 July and 53 mm on 25 August 1999 (Table 18). The harvest started on 11 October 1999 and the average grain yield was about 12.7 t/ha with a grain moisture content of 15%.

### Table 18 Irrigation parameters – Lavalette site, Montpellier (France)

<table>
<thead>
<tr>
<th>Inflow, ( Q_{in} ) [m/s]</th>
<th>Irrigation L1</th>
<th>Irrigation L2</th>
<th>Irrigation L3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0012</td>
<td>0.00073</td>
<td>0.00077</td>
</tr>
<tr>
<td>Cut-off time, ( t_{co} ) [min]</td>
<td>110</td>
<td>160</td>
<td>120</td>
</tr>
</tbody>
</table>

### Table 19 Soil hydraulic characteristics for the Lavalette site, Montpellier (France)

<table>
<thead>
<tr>
<th>Layer \ Soil Parameter</th>
<th>( \theta_s )</th>
<th>( \theta_r )</th>
<th>( \alpha ) [m(^{-1})]</th>
<th>( n )</th>
<th>( K_s ) [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 – 0.55 m</td>
<td>0.35</td>
<td>0.05</td>
<td>1.5</td>
<td>1.457</td>
<td>4.17 \times 10^{-6}</td>
</tr>
<tr>
<td>0.55 – 0.95 m</td>
<td>0.38</td>
<td>0.05</td>
<td>1.3</td>
<td>1.447</td>
<td>1.39 \times 10^{-6}</td>
</tr>
<tr>
<td>0.95 – 2.00 m</td>
<td>0.41</td>
<td>0.05</td>
<td>1.9</td>
<td>1.31</td>
<td>5.17 \times 10^{-7}</td>
</tr>
</tbody>
</table>

### 4.1.2 Single irrigation events

The coupled surface-subsurface flow model FAPS is subsequently tested on various experimental data on single irrigation events. In technical literature, irrigation model simulations have been frequently compared to experimental data, where infiltration characteristics are described by the Kostiakov equation (cf. subsection 4.1.1.1). On the other hand, experimental data for the sufficient parameterization of an entire irrigated growing season, which includes
the soil hydraulic parameters of the VGM model and the distributed measurements of soil moisture, are rarely reported. The FAPS performance is subsequently tested (i) by utilizing the Kostiakov equation (FAPS-K) in order to make it comparable to standard experiments and to simulations by other irrigation models and (ii) by utilizing HYDRUS-2 (FAPS-H). The test of FAPS-H is based on data collected during laboratory experiments at the University of Technology, Dresden, Germany as well as on field experiments at IT Kharagpur, India and at CEMAGREF in Montpellier, France.

4.1.2.1 Prediction indices
A main model performance measure is the comparison of predicted and observed advance and recession times. In order to evaluate how well the coupled model mirrors the measured irrigation advance (and recession) data, five statistical parameters are subsequently used, which are:

(1) the root-mean-square error

\[ \text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (t_{o}^{(i)} - t_{p}^{(i)})^2} \]  \hspace{1cm} (167)

where

\( t_{p} \) and \( t_{o} \) denote the predicted and observed advance (and recession) times respectively and \( N \) is the number of data points used in the evaluation.

(2) the average absolute difference

\[ D_a = \frac{1}{N} \sum_{i=1}^{N} |t_{o}^{(i)} - t_{p}^{(i)}| \]  \hspace{1cm} (168)

(3) the average absolute difference to the mean observed advance (recession) time \( \bar{t}_{o} \) as a percentage

\[ D_{am} = \frac{100}{N} \sum_{i=1}^{N} \left| \frac{t_{o}^{(i)} - t_{p}^{(i)}}{\bar{t}_{o}} \right| \]  \hspace{1cm} (169)

(4) the average absolute error \( E_a \) as a percentage

\[ E_a = \frac{100}{N} \sum_{i=1}^{N} \left| \frac{t_{p}^{(i)} - t_{o}^{(i)}}{t_{o}^{(i)}} \right| \]  \hspace{1cm} (170)

(5) the coefficient of efficiency by Nash-Sutcliff [ASCE, 1993]

\[ C_e = 1 - \frac{\sum_{i=1}^{N} (t_{o}^{(i)} - t_{p}^{(i)})^2}{\sum_{i=1}^{N} (t_{o}^{(i)} - \bar{t}_{o})^2} \]  \hspace{1cm} (171)
4.1.2.2 FAPS model with Kostiakov infiltration

All models within one of the four classes of surface flow equations (cf. subsection 3.3.1.2) should generally converge to the same results [Bassett and Fangmeier, 1980]. This is investigated subsequently by comparing the FAPS-K model results with the results of four other ZI surface irrigation models which are evaluated in Esfandiari [1997]. In addition, FAPS-K is applied to various field experimental data with different field geometry parameters and infiltration characteristics.

FAPS-K versus ZI irrigation models from literature

Esfandiari [1997] evaluated a number of irrigation models which employ the complete hydrodynamic surface flow equations, ZI models and KW models. All the models in this study employ the modified Kostiakov equation. The four ZI models employed are the Walker model [SIRMOD III, 2003], the Strelkoff model [Strelkoff and Katopodes, 1977], the Ross model [Ross, 1986] and the Elliott model (only for the advance phase) [Elliott and Walker, 1982]. The performance of these four models is compared with the performance of FAPS-K using two field experiments of very distinguished characteristics which are described by Esfandiari [1997]. These experiments, namely the WP11 and the FS47 runs (cf. Table 12), are selected because they are examples of a short and a long furrow.

As seen in Table 20, FAPS-K predicts FS47 advance times better than the other models. The average absolute error $E_a$ is 2.2% less than the one of the Strelkoff model and 10.3% less than the one of the Walker model. In addition, the coefficients of efficiency and determination (0.98 and 1.00, respectively) indicate a very good fit of predicted and observed advance times, which is confirmed by the plot in Figure 60. FAPS-K performance for predicting WP11 advance times is also good (Figure 61), again having the smallest $E_a$ value of 12.0% of all the models and having values of $RMSE$ and $Da$ which were similar to those calculated for the other models (Table 21). The Strelkoff model yields no convergent solution for WP11.

The FAPS-K error indices for run WP11 recession are similar in value when compared to the ones of the other models (cf. Table 21). On the other hand, FS47 recession times are not well predicted by FAPS-K (Figures 60 and 61). As seen in Table 20, the $RMSE$ of the FS47 run is much higher than for the other models. Although the coefficient of determination is high, it does not ensure a good model fit. Moreover, the $Ce$ value is negative. According to Hall [2001], $Ce$ is liable to fall below zero when a strong bias is introduced to the computed output or when the model produces relatively large timing errors. The latter is the case for FS47 as can be seen in Figure 60. Beran [1999] states that an efficiency coefficient of 0.95 or more is required to ensure a good model performance (which none of the models in the study achieved in the case of WP11 recession).
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Table 20  Performance criteria for different ZI models in the case of the FS47 run

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE [min]</th>
<th>$D_a$ [min]</th>
<th>$D_{am}$ [%]</th>
<th>$E_a$ [%]</th>
<th>$C_e$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FAPS-K</td>
<td>0.30</td>
<td>0.26</td>
<td>9.3</td>
<td>11.5</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>Ross</td>
<td>0.59</td>
<td>0.49</td>
<td>17.7</td>
<td>20.0</td>
<td>0.91</td>
<td>0.99</td>
</tr>
<tr>
<td>Walker</td>
<td>0.61</td>
<td>0.51</td>
<td>18.5</td>
<td>21.8</td>
<td>0.91</td>
<td>0.99</td>
</tr>
<tr>
<td>Strelkoff</td>
<td>0.54</td>
<td>0.44</td>
<td>16.1</td>
<td>13.7</td>
<td>0.92</td>
<td>0.99</td>
</tr>
<tr>
<td>Elliot</td>
<td>0.62</td>
<td>0.52</td>
<td>19.0</td>
<td>21.6</td>
<td>0.90</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Table 21  Performance criteria for different ZI models in the case of the WP11 run

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE [min]</th>
<th>$D_a$ [min]</th>
<th>$D_{am}$ [%]</th>
<th>$E_a$ [%]</th>
<th>$C_e$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FAPS-K</td>
<td>25.2</td>
<td>18.7</td>
<td>4.8</td>
<td>12.0</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>Ross</td>
<td>26.0</td>
<td>20.0</td>
<td>5.2</td>
<td>17.4</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>Walker</td>
<td>25.4</td>
<td>19.5</td>
<td>5.0</td>
<td>14.9</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>Strelkoff</td>
<td>26.0</td>
<td>20.0</td>
<td>5.2</td>
<td>17.4</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>Elliot</td>
<td>25.4</td>
<td>19.5</td>
<td>5.0</td>
<td>14.9</td>
<td>0.99</td>
<td>1.00</td>
</tr>
</tbody>
</table>

FAPS-K versus SRFR

The model SRFR is a public-domain software based on the Strelkoff model and was developed at the US Department of Agriculture, Agriculture Research Service, U.S. Water Conservation Laboratory, Phoenix [Strelkoff, 1991] (cf. subsection 1.3.1). Since the model is widely tested and shows good results for predicting advance and recession times (cf. e.g. Esfandiari [1997]), FAPS-K results are compared to the SRFR output for another two experiments from US sites in Colorado, namely Matchett 2-3-5 and Printz 3-2-3 (cf. Table 12).

SRFR employs parameters for the hydraulic section which are different from the ones used in FAPS. Because of this, a methodology is developed to derive the SRFR parameters $a$ and $b$ of the geometry function

$$y(h_w) = \left( \frac{h_w}{a} \right)^{1/b}$$

(173)
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Equation 69 yields

\[
21\frac{1}{p_wlA} = \left( \frac{w}{p_1} \right)
\]  

(174)

SRFR parameters \( a \) and \( b \) are not accessible directly from Equation 174 and, thus, are determined iteratively as follows:

**Determining SRFR parameters \( a \) and \( b \)**
- The FAPS parameters \( p_1 \) and \( p_2 \) are given in Equation 174.
A vector of equally spaced flow-depth values is defined: 
\[ H_{wl} = \{ h^{(j)}_{wl} \} = \{ 0.0, 0.01, \ldots, 0.020 \} \text{ m}, \]  
where \( j = 1 \ldots 21 \).

- Corresponding cross-sectional area vectors \( A_{FAPS} \) are calculated for all elements of \( H_{wl} \), using Equation 174.
- Starting values/initial guess of parameters: \( a_i = 0.01 \) and \( b_i = 1.0 \).
- Using \( a_i = 0.01 \) and \( b_i = 1.0 \) in Equation 173 and integration with respect to flow depth (using Simpson's integration) yields the vector of cross-sectional area \( A_{SRFR} \) corresponding to \( H_{wl} \):

\[
A_{SRFR}^{(j)} = \int_{y^{(j)}}^{y^{(j+1)}} h^{(j)}_{wl} \Delta y \quad \text{for all } j
\]  
where \( \pm y^{(j)} = \pm h^{(j)}_{wl} \) (cf. Equation 173).

- With the variables \( a \) and \( b \), the differences between vector elements of \( A_{FIM} \) and \( A_{SRFR} \) are minimized by using the Matlab function \textit{fminsearch} (multidimensional unconstrained nonlinear minimization (Nelder-Mead)). The objective function reads

\[
\sum_{j=1}^{21} \left( A_{FIM}^{(j)} - A_{SRFR}^{(j)} \right)^2 \rightarrow \text{Minimum}
\]  

If subsequent iterations of Equation 176 meet the given precision criteria, the program is terminated with final values of \( a \) and \( b \).

Following the above approach for the Matchett 2-3-5 input data (Table 12), the SRFR input parameters are derived as \( a = 0.002477 \) and \( b = 2.176 \) for the cross-sectional profile function Equation 173, where \( y \) is given in [mm] as required by the program SRFR.

Both FAPS and SRFR are applied on the Matchett 2-3-5 data set. Calculated FAPS-K advance times are almost identical to those calculated by SRFR as seen in Figure 62. Simulated recession times, however, differ to about 12 s, due to differences between the two models in describing surface flow during the recession phase.

### More FAPS-K simulations

FAPS was tested on a large number of other experimental data sets from the US sites in Colorado, Utah, Idaho and two Australian sites (see subsection 4.1.1.1). Three experiments have been selected for illustration here. The Flowell wheel experiment (Table 12) is characterized by a low inflow/infiltration ratio, which results in increasingly low advance rates as seen in Figure 64. A similar field length of 350 m but a higher inflow rate (3.49 l/s) characterizes the Printz 3-2-3 experiment, whereas the Matchett 2-3-5 run was conducted on a 425 m long field with a low discharge of 0.92 l/s. All selected experiments were conducted on different soil types as seen in Table 12.

FAPS-K shows a good match of predicted and observed advance times (Figures 62, 63, 64), which was confirmed by high \( R^2/C_e \) values of 1.0/0.91, 0.98/0.90 and 1.0/0.94 for the Flowell wheel, Printz 3-2-3 and Matchett 2-3-5 run, respectively (Table 22). Recession times for these experiments, however, are under-predicted by FAPS. \( RMSE \) and \( D_a \) values as well as \( E_a \) and
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Dam values are generally lower than the corresponding values for predicted advance times. But the $C_v$ values are again below zero (Table 22) and indicate large timing errors.

---

2 Exception: $RMSE$ for Printz 3-2-3 recession is higher than $RMSE$ for advance.
Irrigation control: towards a new solution of an old problem

Table 22 Performance indices for predicted FAPS-K advance and recession times

<table>
<thead>
<tr>
<th>Performance index</th>
<th>Matchett 2-3-5</th>
<th>Printz 3-2-3</th>
<th>Flowell wheel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Advance</td>
<td>Recession</td>
<td>Advance</td>
</tr>
<tr>
<td>RMSE [min]</td>
<td>20.6</td>
<td>10.7</td>
<td>6.1</td>
</tr>
<tr>
<td>Da [min]</td>
<td>18.8</td>
<td>9.9</td>
<td>5.0</td>
</tr>
<tr>
<td>Dam [%]</td>
<td>15.7</td>
<td>0.7</td>
<td>15.9</td>
</tr>
<tr>
<td>Ea [%]</td>
<td>23.0</td>
<td>0.7</td>
<td>21.4</td>
</tr>
<tr>
<td>Ce</td>
<td>0.94</td>
<td>-0.24</td>
<td>0.90</td>
</tr>
<tr>
<td>R²</td>
<td>1.00</td>
<td>0.87</td>
<td>0.98</td>
</tr>
</tbody>
</table>

4.1.2.3 FAPS model with HYDRUS-2D

Both laboratory and field experimental data have subsequently been used to test the FAPS-H model. Predicted advance and recession times have been compared to observations. In addition, pressure head distribution predicted by the 2D subsurface model HYDRUS-2D is compared to tensiometer measurements of the laboratory experiments.

FAPS-H simulation of laboratory experiments

Irrigation experiments were conducted at the Hubert-Engels Laboratory at the Institute of Hydraulic Engineering and Applied Hydromechanics, Dresden University of Technology, as described in subsection 4.1.1.2. The soil hydraulic parameters of the VGM model are determined by the inverse method. FAPS-H is first tested on the basis of irrigation advance data from the first experimental run. Then, FAPS-H model results are compared with the outcome of experimental runs two and three, which are not involved in estimating the soil hydraulic parameters.

Input parameters and boundary conditions for FAPS-H simulations corresponding to these irrigation experiments are listed in Table 13. Manning's roughness coefficient of the furrow is estimated as $n = 1/K_x = 0.03$ from velocity measurements and calibration using run one. This value is used for the simulation of all experimental runs. HYDRUS-2D initial conditions...
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are provided by pressure head readings of the tensiometer probes averaged over all the five cross sections (Table 14).

Figures 65, 66, 67 and Table 23 present the measured and simulated irrigation advance times for the three experimental runs. The model test (run 1) shows perfect match of the observed and simulated advances, which is confirmed by \( \text{RMSE} \), \( R^2 \) and \( C_a \) values of 0.06 min, 0.99 and 0.98, respectively. A similar agreement is achieved for the fifth irrigation experiment, where the performance criteria \( \text{RMSE} \), \( R^2 \) and \( C_a \) attains values of 0.04 min, 1.0 and 0.99, respectively. For the fourth irrigation, however, simulated advance is faster than experimental advance. This can be explained by the fact that there was an eleven-week interval between this and the preceding experiment, which resulted in a cracking, albeit moderate, of the soil and consequently a significant increase in the initial infiltration rate. These phenomena were, of course, overlooked and not taken into account by the mathematical model. Consequently, \( \text{RMSE} \) and \( C_a \) values are relatively poor, at 0.2 min and 0.82, respectively.

![Figure 65](image1.png)

Figure 65  Observed and predicted advance times of the laboratory experiments, run 1

![Figure 66](image2.png)

Figure 66  Observed and predicted advance times of the laboratory experiments, run 4
Figure 68 shows a comparison of simulated and measured matric heads of run one at $x_{\text{inf}} = 6.3$ m and $x_{\text{inf}} = 18.3$ m, after 20 mins irrigation time. The simulation compares favourably with the measurements although deviations are observed at specific points. This might be due to discrepancies between the local measurements of pressure head and the values actually taken for initializing the simulation, namely, the averaged pressure heads between the cross sections, linearized over the depth.

Table 23 Performance indices of the laboratory experiments

<table>
<thead>
<tr>
<th>Performance</th>
<th>Experiment 1</th>
<th>Experiment 4</th>
<th>Experiment 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE [min]</td>
<td>0.06</td>
<td>0.20</td>
<td>0.04</td>
</tr>
<tr>
<td>$D_{am}$ [min]</td>
<td>0.05</td>
<td>0.19</td>
<td>0.03</td>
</tr>
<tr>
<td>$E_a$ [%]</td>
<td>9.1</td>
<td>21.8</td>
<td>13.8</td>
</tr>
<tr>
<td>$D_{am}$ [%]</td>
<td>4.2</td>
<td>15.7</td>
<td>4.5</td>
</tr>
<tr>
<td>$C_v$</td>
<td>0.98</td>
<td>0.82</td>
<td>0.99</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

FAPS-H simulation of Kharagpur field experiments
Irrigation experiments were conducted at the experimental site of IIT Kharagpur, India, as described in subsection 4.1.1.3. Input parameters, the initial conditions and the boundary conditions can be found in Tables 15-17 of the same subsection.

Figure 69 presents the measured and simulated irrigation advance and recession times for experimental runs 1 and 5. Both the simulated advance times and the recession times of run 5 match perfectly with the observations which is confirmed by RMSE, $R^2$ and $C_v$ values of 0.3/1.2 min, 1.00/1.00 and 0.92/0.74, respectively (cf. Table 24).
Even the first run, where soil was freshly prepared and therefore not yet stable in structure, shows an excellent match between the observed and simulated advances, which is confirmed by RMSE, $R^2$ and $C_o$ values of 2.3 min, 0.99 and 0.7, respectively. The recession times of this run are predicted with a relatively small RMSE of 1.7 min. However, a total of two data points out of four observed recession times are poorly predicted by FAPS-H as seen in Figure 69. Consequently, both the coefficients of efficiency and determination are small. Nevertheless, this must not be overrated for the following reasons. Physically, one would not
expect a slow recession within the first quarter of the field length preceding a comparatively fast recession on the remaining three quarters of the field (as observed). Difficulties in measuring field data, recession times in particular, are frequently reported in literature (e.g. Esfandiari and Maheshwari [2001]). Moreover, it should be noted that four data points are not sufficient, and therefore irrelevant, for a significant statistical analysis.

The total infiltrated volume of irrigation 1 was calculated by FAPS-H to be 0.461 m³. This tallies well with the observed value of 0.452 m³ from inflow/outflow measurements.

**FAPS-H simulation of Lavalette field experiments**

Comprehensive irrigation experiments were conducted at the experimental site of CEMAGREF in Montpellier, France, as described in subsection 4.1.1.4. Since the relevant furrow irrigation experiments were conducted in closed end furrow (CEF) irrigation technique, only irrigation advance simulated by FAPS-H is compared with the observations.

For the furrow irrigation runs L2 and L3, the mean of the observed advance times from the 30 furrows is presented in Figure 70 together with the corresponding FAPS-H simulations. The fastest and slowest observed advance is added to the plots in broken lines. It can be seen that FAPS-H generally predicts advance times which are much too fast. This is confirmed by poor RMSE and Ce values as seen in Table 25. Because the physical limit of FAPS-H simulated infiltration is well below the infiltrated volume measured in the field, the simulated advance times of these runs, however, cannot match with the observation if using the soil hydraulic parameters and the parameterization of irrigation L2 and L3 as given in subsection 4.1.1.4. To underline this statement, a simple volume balance approach is subsequently applied to analyze infiltration during irrigation L3:

At the time, when the irrigation advance has just reached the lower field end, the volume balance can be roughly estimated as being...
where \( \bar{A} \) denotes the mean cross-sectional area, \( wp \) is the corresponding mean wetted perimeter and \( K_s \) is the saturated conductivity of the first soil layer. The first and second term on the left-hand side of Equation 177 denotes the inflow volume and the volume of water in the furrow respectively, assuming a horizontal (mean) water level \( \bar{h} \). The right-hand side of Equation 177 is an estimate of the maximum possible infiltration volume assuming the total field length flooded from the beginning of the water application (thus neglecting the advance process and overestimating infiltration opportunity time and infiltration volume). In the case of irrigation L3, the mean observed flow depth was 0.05 m and, thus, \( \bar{A} \) and \( wp \) is calculated to be \( 7.44 \times 10^{-3} \) m² and 0.249 m, respectively. These values together with \( K_s = 7.7 \times 10^{-7} \) m/s, \( x_L = 130 \) m and \( t_s = 4740 \) s (mean) inserted in Equation 177 leads to:

\[
Q_0 \cdot t_s - \bar{A} \cdot x_L = wp \cdot x_L \cdot K_s \cdot t_s
\]

(177)

and

\[
3.65 - 0.97 > 1.17
\]

(179)

According to this volume balance calculation, the infiltrated volume (inflow volume minus volume in the furrow) must be about 2.63 m³ at the time \( t = t_s \). In contrast, the maximum possible infiltration for the given parameterization is less than 1.17 m³.
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Table 25  Performance indices of the Lavalette field experiments

<table>
<thead>
<tr>
<th>Performance</th>
<th>Irrigation L2</th>
<th>Irrigation L3</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE [min]</td>
<td>37.5</td>
<td>16.8</td>
</tr>
<tr>
<td>$D_{a1}$ [min]</td>
<td>31.9</td>
<td>15.2</td>
</tr>
<tr>
<td>$E_{a1}$ [%]</td>
<td>64.1</td>
<td>48.9</td>
</tr>
<tr>
<td>$D_{a2}$ [%]</td>
<td>60.4</td>
<td>41.3</td>
</tr>
<tr>
<td>$C_{e1}$</td>
<td>-0.09</td>
<td>0.52</td>
</tr>
<tr>
<td>$R^2$</td>
<td>1.00</td>
<td>0.99</td>
</tr>
</tbody>
</table>

It can be concluded from the above analysis that the initial water flux into the soil (i.e. shortly after wetting) is higher than the given $K_s$ value. Consequently, the parameterization of the soil model does not represent the actual soil hydraulics of the considered field experiments.

The Lavalette experiments illustrate the limitations connected with the mechanistic modelling approach, especially those of the Richards equation (cf. subsection 3.3.1.3).

4.1.2.4 Simulating single irrigation events – results

The simulation of various irrigation experiments from literature by FAPS-K shows a perfect match between simulated and observed irrigation advance times. A similarly good match is achieved between predicted FAPS-H advance times and the observations of the laboratory experiments (Dresden) and the Kharagpur field experiments. The results of laboratory experiment run four, however, indicated that soil cracking, due to a relatively high clay content combined with a relatively long drying period prior to this irrigation experiment, had a considerable impact on the infiltration process and, consequently, on irrigation advance, which later was confirmed by tracer experiments on the same laboratory setup [Rawson, 2001].

In the case of the Lavalette field experiments, FAPS-H overestimates the advance process. As confirmed by simple volume-balance calculations, the soil hydraulic parameters must lead to an underestimation of the short-time infiltration (e.g. infiltration shortly after wetting) in the coupled model FAPS-H. It is again emphasized that the realistic estimation of the soil hydraulic parameters plays a key role in accurate furrow irrigation modelling.

Although simulated recession times match favourably with observations of the Kharagpur field experiments, FAPS significantly overestimates recession times in most other applications. This is likely to be caused by the simplification of the surface flow equations in the depletion/recession module which may need further development.

The physically based simulation of the subsurface flow is still a challenging task. The spatially high variability of soil hydraulic characteristics, changes in soil structure especially due to settlements between first and second irrigation application and those due to cracking soils can lead to a high variability of infiltration characteristics in space and time. These conditions and phenomena cannot be described by the physically based model HYDRUS-2D because (i) highly distributed soil hydraulic parameters of soil planes are currently not accessible on field level and (ii) the Richards equation is not valid for deformable, cracking soils. Furthermore, the soil model parameterization can be erroneous because local measurements of soil moisture and/or matric head in highly variable soils do often not represent the 'real' or average conditions of a soil plane, not to mention of the total field. This well-known scaling problem can cause significant errors not only by inverse parameter
estimation but also by a poor estimation of initial conditions from local measurements and by comparing calculated and observed soil moisture.

FAPS-H currently employs soil variability in the soil plane (e.g. soil layers), but assumes both the soil hydraulic properties and the initial conditions to be invariant along the furrow length. For simulations of the Kharagpur and Lavalle field experiments (where only a limited number of tensiometer measurements were available) the locally measured matric head differs significantly from the simulated matric head during the irrigation, which supports the above statements.

In the case of the laboratory experiments, however, where soil hydraulic properties can be estimated carefully by using the inverse solution technique together with a large number of local measurements in the soil plane, the soil moisture distribution can also be sufficiently well simulated for a well-pronounced dynamic flow during irrigation.

In order to employ soil variability and/or variable initial conditions along the furrow in FAPS-H, the present version of the model can be modified with comparatively little effort. But since this modification significantly increases the parameterization effort (⇒ input data requirements), it is not considered within the framework of this contribution.

Summarizing the results of this subsection, the coupled irrigation model FAPS-H, besides providing a detailed insight into the interacting surface-subsurface flow during the irrigation process, yields reliable results provided that the soil hydraulic parameters are carefully derived. FAPS-H shows a significant potential for improving furrow irrigation design and management.

### 4.1.3 Growing season

Most experimental data on furrow irrigation reported in technical literature cover single irrigation events only, are based on empirical infiltration parameters and do not provide data on crop growth (cf. subsection 4.1.1.1). On the other hand, high resolution data (in both space and time) of the soil water transport were obtained from laboratory experiments. Unfortunately, the growing of the grass crop was not successful in the laboratory (cf. subsection 4.1.1.2). Soil moisture measurements were not as comprehensive during the Lavalle field experiments in France as during the laboratory experiments, but crop growth was nevertheless monitored on various dates during the 1999 growing season (cf. subsection 4.1.1.4). For this reason the Lavalle experimental data were the most suitable data for the FIM model validation. It should be noted that FIM was designed for free-draining surface flow conditions (FDF) but the three irrigations at Lavalle were conducted by the closed-end furrow method (CEF). However, the two upstream monitoring sections of the field (at the inlet location and at $x_{\text{inf}} = 32.5$ m) were not affected by backwater effects arising from the CEF practice. Because of this, observed soil water content and crop growth data can be compared with FIM simulations at these sections.

The model system FIM is subsequently applied to the Lavalle data since no similar detailed FDF experimental data were available. The simulation of the 1999 growing period at the Lavalle site is subsequently referred to as the Lavalle run.

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3 The field slope of the Lavalle plot is 0.25%.
4.1.3.1 FIM parameterization

The irrigation schedule of the 1999 growing season is listed in Table 26. During the 138 days of corn elevation, the Ta-plot was irrigated only three times due to otherwise sufficient rainfall events. About 70 mm of water were applied between the day of sowing and the middle of June by altogether four sprinkler irrigation events. Mathematically, the sprinkler irrigation events were treated, like precipitation, as a flux across the soil surface in FIM.

<table>
<thead>
<tr>
<th>Days after sowing</th>
<th>Date</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>26/05/1999</td>
<td>Sowing of 11,500 corn seeds/ha</td>
</tr>
<tr>
<td>2-3</td>
<td>28-29/05/1999</td>
<td>23 mm sprinkler irrigation</td>
</tr>
<tr>
<td>5-6</td>
<td>31/05-01/06/1999</td>
<td>22 mm sprinkler irrigation</td>
</tr>
<tr>
<td>20</td>
<td>15/06/1999</td>
<td>14 mm sprinkler irrigation</td>
</tr>
<tr>
<td>21</td>
<td>16/06/1999</td>
<td>11 mm sprinkler irrigation</td>
</tr>
<tr>
<td>45</td>
<td>10/07/1999</td>
<td>1st furrow irrigation</td>
</tr>
<tr>
<td>57</td>
<td>22/07/1999</td>
<td>2nd furrow irrigation</td>
</tr>
<tr>
<td>91</td>
<td>25/08/1999</td>
<td>3rd furrow irrigation</td>
</tr>
<tr>
<td>137</td>
<td>11/10/1999</td>
<td>Harvest of 104,000 plants/ha</td>
</tr>
</tbody>
</table>

Five locations of infiltration computation were considered along the 130 m long furrow at $x_{ref} = [0, 32.5, 65.0, 97.5, 130]$ m, respectively. The subsurface flow model HYDRUS-2D was employed at each location. Figure 71 shows its flow domain geometry which was given by the furrow spacing $f_s = 0.8$ m (width), the soil depth $z_s = 2.0$ m and a parabolic furrow shape derived from on-site measurements (cf. subsection 4.1.1.4). Additionally, Figure 71 portrays...
the calculation mesh and the initial boundary type of the mesh nodes. The cross-sectional area of the subsurface domain was calculated as \( A = 1.563 \text{ m}^2 \).

The **initial soil moisture distribution** on the day of sowing was not accessible directly since rigorous on-site measurements of volumetric water content started on 21 June 1999 at the furrow ridges only (26 days after sowing) and, one month later, at the furrow beds. These neutron probe measurements were taken at \( x = [20, 65, 110 \text{ m}] \) and \( z = [-0.1, -0.2, ... -1.4 \text{ m}] \) at irregular time intervals which varied between two days and two weeks. The observed volumetric water content was representative of the access-tube vicinity only. Functions of mean soil moisture over soil depth \( \theta_w(z) \) were calculated and the difference between the level of the furrow and the level of the ridge was taken into account. 21 July 1999 (56 days after sowing) was the first date of corresponding soil moisture measurements at both furrow bed and ridge.

Although, as mentioned above, the initial soil moisture had not been taken on the day of sowing, it had however been checked at a plot of the Lavalette site as early as 1 May 1999, i.e. 26 days before sowing. On this date the average soil moisture was about \( \theta_w \approx 0.2 \text{ m}^3/\text{m}^3 \) up to a depth of 1.2 m. A reasonable option to obtain the initial soil moisture distribution of the Lavalette run (at the day of sowing) from these earlier measurements was the forward simulation by HYDRUS-2D. Hence, a simulation of soil moisture redistribution during these 26 days (1 to 26 May 1999) was conducted by HYDRUS-2D using the soil moisture measurements on 1 May 1999 for initialization and the observed (time-variable) meteorological boundary conditions. The values of the simulated soil moisture content in the upper soil layer were slightly increased in order to match with the observed soil water storage on 21 July 1999.

Figure 72 shows the final soil moisture distribution which was taken for the initialization of the Lavalette run.
The atmospheric boundary conditions of the subsurface flow model HYDRUS-2D were calculated from the meteorological data and shown in Figure 73. Altogether 245.5 mm precipitation were observed during the growing season as seen in Figure 73, but only a total of 114 mm fell during the vegetative period from June to the end of September.

During times of irrigation, the flow depth is calculated by the surface flow model FAPS as described in subsections 3.3.5 and 3.3.6. Input parameters and boundary conditions for the simulation of the three irrigation events by the surface flow model FAPS are given in subsection 4.1.1.4 and Table 18.

Horizontally, the subsurface flow domain was structured by three soil layers. The soil hydraulic properties of these layers were described by the VGM model parameters which are listed in Table 19. An initial FIM run with this soil parameterization resulted in systematically too little simulated infiltration. For example, the total simulated infiltrated volume of irrigation L3 was simulated as $I_{inf} = 34 \text{ mm } [10^{-3} \text{m}^3/\text{m}^2]$ at $x_{inf} = 32.5 \text{ m}$, which was less than the observed volume of 50 mm. A logical consequence of underestimated infiltration (especially for infiltration times shortly after wetting) were overestimated irrigation advance rates as discussed in more detail in subsection 4.1.2. It can be concluded that the soil

4 The under-prediction of the infiltrated water volume results in an under-prediction of plant-available soil water and consequently to lower crop yield.
parameters (Table 19) do not represent the real soil properties of the Lavalette site and, thus, more realistic soil hydraulic parameters are required for a better mapping of the infiltrated volume. A rather simple, but reasonable, option for satisfying this requirement is to increase the values of saturated hydraulic conductivity of the first two soil layers as shown in Table 27. For the irrigation event L3, this modification resulted in a simulated infiltrated volume of \( I_{\text{inf}} = 51 \text{ mm} \) at \( x_{\text{inf}} = 32.5 \text{ m} \), which was about the same size as the observed value. Subsequently, the modified soil hydraulic parameters (Table 27) are used for the simulation of the Lavalette run.

Table 27 Modified soil hydraulic parameters of the Lavalette plot

<table>
<thead>
<tr>
<th>Layer \ Soil Parameter</th>
<th>( \theta_s )</th>
<th>( \theta_r )</th>
<th>( \alpha \text{ [m}^{-1}] )</th>
<th>( n )</th>
<th>( K_s \text{ [m} s^{-1}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 – 0.55 m</td>
<td>0.35</td>
<td>0.05</td>
<td>1.5</td>
<td>1.457</td>
<td>( 7.5 \cdot 10^{-6} )</td>
</tr>
<tr>
<td>0.55 – 0.95 m</td>
<td>0.38</td>
<td>0.05</td>
<td>1.3</td>
<td>1.447</td>
<td>( 1.85 \cdot 10^{-6} )</td>
</tr>
<tr>
<td>0.95 – 2.00 m</td>
<td>0.38</td>
<td>0.05</td>
<td>1.9</td>
<td>1.310</td>
<td>( 5.17 \cdot 10^{-7} )</td>
</tr>
</tbody>
</table>

4.1.3.2 FIM simulation

The total CPU time consumption of a FIM simulation depends on a large number of factors. It increases with the number of simulated days, the number of irrigation events, the number of computational knots where the infiltration is calculated, and the size (and resolution) of the subsurface calculation mesh. It also increases if strong soil moisture/matric head gradients occur during the simulation. A low inflow rate during irrigation also leads to higher computation time as compared to higher inflow rates since the simulation of the irrigation advance phase is much more time-consuming than the simulation of the storage phase (where no iteration is required). The computation is additionally prolonged if the atmospheric boundary conditions frequently change, e.g. due to rainfall events.

Of course, the CPU time consumption is dependent upon the CPU type of the computer which is used for the simulation. Table 28 shows the CPU time requirement of the FIM simulation of the Lavalette run for three PCs with different computing power. The simulation with the 'symmetric furrow' option, where the subsurface domain is halved in size, required an average of only 61% of the computing time of the full subsurface domain. If no additional rainfall events were considered, the computing time went down to an average of 40%.

Table 28 CPU time requirements for the simulation of the Lavalette run by FIM

<table>
<thead>
<tr>
<th>Computer type</th>
<th>Simulated cross section</th>
<th>CPU time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMD Athlon XP 1600+, 0.5GB RAM</td>
<td>whole</td>
<td>1067.3</td>
</tr>
<tr>
<td>AMD Athlon XP 2400+, 0.5GB RAM</td>
<td>whole</td>
<td>777.2</td>
</tr>
<tr>
<td>Pentium M755, 2.0 GHz, 0.5GB RAM</td>
<td>whole</td>
<td>544.7</td>
</tr>
<tr>
<td>AMD Athlon XP 1600+, 0.5GB RAM</td>
<td>half</td>
<td>654.9</td>
</tr>
<tr>
<td>AMD Athlon XP 2400+, 0.5GB RAM</td>
<td>half</td>
<td>459.7</td>
</tr>
<tr>
<td>Pentium M755, 2.0 GHz, 0.5GB RAM</td>
<td>half</td>
<td>346.4</td>
</tr>
<tr>
<td>AMD Athlon XP 2400+, 0.5GB RAM</td>
<td>half, no rainfall</td>
<td>286.1</td>
</tr>
<tr>
<td>Pentium M755, 2.0 GHz, 0.5GB RAM</td>
<td>half, no rainfall</td>
<td>233.4</td>
</tr>
</tbody>
</table>
It should be noted that FIM can be adapted to parallel processing which would further reduce the CPU time requirements by the estimated factor 2 – 4 (provided that the FIM is running on a multi-processor computer system).

4.1.3.3 FIM simulation – results and discussion

The simulation results of the Lavalette run by FIM are presented in this section and compared to observations. First of all, the simulation of the irrigation advance times and the irrigation performance measures of the three irrigation events are presented and discussed. In the following subsections, the overall water balance, the soil water storage during the entire growing season and the soil moisture content are shown. The impact of the irrigation method (FDF/CEF) on infiltration is discussed in another subsection by analyzing the simulated and observed soil water storage during the growing season. Furthermore, the overall irrigation efficiency is calculated. Finally, the simulated evapotranspiration, the leaf-area index and the final crop yield is presented.

Irrigation advance and irrigation performance

Irrigation advance times of the second and third irrigation event were observed at 30 furrows and the advance data was analyzed statistically [Mailhol, 2001]. The advance times of the first irrigation event (L1) were monitored at a few furrows only. In addition, the soil hydraulic parameters of L1, which were most likely to be different from those of the following irrigation events due to the instable soil of freshly corrugated furrows, were not known. For this reason, the L1 event is not included in the analysis.

Figure 74 shows the simulated and observed irrigation advance times of the second and third irrigation events. FIM was underestimating advance times for both irrigations even when using the modified soil hydraulic parameters (Table 27). In FIM, the irrigation advance rate is directly correlated to the infiltrated water volume. A simulated advance rate which is higher than the observed one leads consequently to an infiltration volume which is lower than the observed one. In the case that the total simulated infiltration volume (i.e. the infiltration volume of all irrigation phases) correlates with the observed volume and the corresponding advance rates do not match, this means that the subsurface flow dynamics are not mapped.

Figure 74 Simulated and observed irrigation advance times of the Lavalette runs L1, L2 and L3
correctly by the model. As already discussed in subsection 4.1.2, more realistic soil hydraulic parameters were required to map both the relatively high infiltration rates for the short opportunity times and the low infiltration rates for the long opportunity times which were observed at the Lavalette plot.

Four irrigation performance measurements, namely, the irrigation efficiency IE, the application efficiency AE, the low-quarter distribution uniformity $DU_{lq}$ and the low-quarter adequacy $AD_{lq}$ were calculated for each of the three irrigation events and listed in Table 29. Some measures which were required for the computation of the performance criteria are shown in Table 30. IE and AE of irrigation L1, L2 and L3 were computed for the time intervals [44 - 54] das, [55 - 88] das and [89 - 137] das, respectively. The irrigation efficiency was high for the irrigation events L1 and L2 because the percentage of irrigation water on the total evapotranspiration was relatively low (on average only 27.3%). The IE of irrigation L2 was especially high because $ET_{a,c}$ was high, runoff was low and the irrigation water compensated the crop requirements which were not covered by precipitation, capillary rise and the change in soil water storage as seen in Tables 29 and 30. On the other hand, the IE of L3 was zero because the $ET_{a,c}$ was almost fully compensated by precipitation (Table 30). The capillary rise added to the remaining part. No irrigation water was beneficially used and the irrigation L3 would not have been necessary.

Table 29 Simulated irrigation performance measures of the Lavalette irrigation events L1, L2 and L3

<table>
<thead>
<tr>
<th>Irrigation event</th>
<th>L1</th>
<th>L2</th>
<th>L3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IE$ [%]</td>
<td>77.7</td>
<td>95.9</td>
<td>0.00</td>
</tr>
<tr>
<td>$AE$ [%]</td>
<td>67.9</td>
<td>83.5</td>
<td>83.6</td>
</tr>
<tr>
<td>$DU_{lq}$</td>
<td>0.92</td>
<td>0.79</td>
<td>0.79</td>
</tr>
<tr>
<td>$AD_{lq}$</td>
<td>0.72</td>
<td>0.53</td>
<td>0.27</td>
</tr>
</tbody>
</table>

In the context of this study, the application efficiency AE is a measure of how much of the irrigation water contributes towards compensating the SMD. It is correlated to the runoff: if the runoff is high compared to the total volume applied to the field, then the AE is low (L1) and vice versa (L2 and L3), as shown in Table 29.

The values of $AD_{lq}$ in Table 29 indicate 28% under-irrigation of L1 ($AD_{lq} = 0.72$), 47% under-irrigation of L2 ($AD_{lq} = 0.53$) and 73% under-irrigation of L3 ($AD_{lq} = 0.27$). But rainfall events ended up providing almost optimal crop water supply during the entire growing season (cf. Figure 73).

The low-quarter distribution uniformity for irrigation L1 (0.92) was higher than for both L2 and L3 (0.79) because the $DU_{lq}$ is correlated to the inflow rate $Q_0$, which was higher for L1 ($1.23 \times 10^3$ m$^3$/s) than for the other two irrigation events ($0.73 \times 10^3$ m$^3$/s and $0.77 \times 10^3$ m$^3$/s, respectively).
Irrigation performance criteria may vary significantly with the irrigation method, e.g. if FDF or CEF is applied. As already mentioned, FIM simulates FDF but CEF was applied during the experiments. A difference between the simulated and the observed soil water flow-regime is expected at the tail end section of the field due to the backwater effect of the CEF practice (cf. subsection 4.1.3.3). The values of DU and $lq_{AD}$ may be particularly affected by this difference.

**Table 30** Measures for calculating the performance criteria of the irrigation events

<table>
<thead>
<tr>
<th>Irrigation event</th>
<th>L1</th>
<th>L2</th>
<th>L3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation day</td>
<td>45</td>
<td>57</td>
<td>91</td>
</tr>
<tr>
<td>Average $\theta$ in the root zone [m$^3$ m$^{-3}$]</td>
<td>0.152</td>
<td>0.151</td>
<td>0.129</td>
</tr>
<tr>
<td>Root zone depth [m]</td>
<td>0.78</td>
<td>0.96</td>
<td>1.20</td>
</tr>
<tr>
<td>Cross sectional area of the root zone [m$^2$]</td>
<td>0.585</td>
<td>0.732</td>
<td>0.922</td>
</tr>
<tr>
<td>Water volume required to compensate SMD in the root zone (at the date of irrigation) [m$^3$]</td>
<td>6.77</td>
<td>8.48</td>
<td>13.3</td>
</tr>
<tr>
<td>Inflow water volume [m$^3$]</td>
<td>8.11</td>
<td>7.01</td>
<td>5.54</td>
</tr>
<tr>
<td>Infiltrated water volume [m$^3$]</td>
<td>5.42</td>
<td>5.94</td>
<td>4.66</td>
</tr>
<tr>
<td>Runoff [m$^3$]</td>
<td>-2.57</td>
<td>-1.07</td>
<td>-0.88</td>
</tr>
<tr>
<td>Precipitation [m$^3$]</td>
<td>0.00</td>
<td>8.22</td>
<td>12.95</td>
</tr>
<tr>
<td>Change in water storage in the root zone [m$^3$]</td>
<td>-2.12</td>
<td>-2.60</td>
<td>6.51</td>
</tr>
<tr>
<td>Capillary rise (toward the root zone) [m$^3$]</td>
<td>0.41</td>
<td>1.11</td>
<td>1.19</td>
</tr>
<tr>
<td>$EI_{c,0}$ for the time interval between successive irrigations [m$^3$]</td>
<td>-8.85</td>
<td>-18.65</td>
<td>-12.70</td>
</tr>
</tbody>
</table>

Irrigation performance criteria may vary significantly with the irrigation method, e.g. if FDF or CEF is applied. As already mentioned, FIM simulates FDF but CEF was applied during the experiments. A difference between the simulated and the observed soil water flow-regime is expected at the tail end section of the field due to the backwater effect of the CEF practice (cf. subsection 4.1.3.3). The values of DU and $AD_{eq}$ may be particularly affected by this difference.

**Water balance and water storage**

Table 31 shows the values of the overall water balance components of the Lavalette run. Altogether 59.0 m$^3$ water were required for crop evapotranspiration. This water was provided mainly by rainfall events (56%), by irrigation (27%) and by the water which was stored in the soil (17%). The total volume of irrigation water was 20.7 m$^3$. A percentage of 16.1 m$^3$ (88%) of this water infiltrated into the soil and 4.5 m$^3$ (22%) left the field by surface runoff (Table 31). The loss due to percolation into deeper soil layers was relatively low (0.6 m$^3$).

The relative mass balance error for the entire FIM simulation was $\varepsilon_{rel} = 1.2\%$.

Table 31 also presents the volume balance components at three (out of five) cross sections, namely, at the field inlet, at the field outlet and at the section $x_{inf} = 32.5$ m. The unit of these components is 10$^3$ m$^3$ per unit surface area (m$^2$) or [mm].

The simulated infiltration decreased from the field head to the tail end (Table 31) as a consequence of the irrigation advance process which lead to a decreasing infiltration opportunity time from field head to tail.

The field inlet and the section $x_{inf} = 32.5$ m were considered untainted with the backwater effects of the CEF irrigation practice. Both the actual transpiration and the actual evaporation at these sections were close to their potential values (Table 31) which indicates that the plants were supplied sufficiently with water during the whole growing period.
Table 31  Total water balance components of the Lavalette run

<table>
<thead>
<tr>
<th>Water balance components</th>
<th>Total field [m³]</th>
<th>Water balance components</th>
<th>x_{inf} = 0 m [mm]</th>
<th>x_{inf} = 32.5 m [mm]</th>
<th>x_{inf} = 130 m [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial soil water content</td>
<td>53.6</td>
<td>Potential transpiration</td>
<td>-345.3</td>
<td>-343.1</td>
<td>-320.1</td>
</tr>
<tr>
<td>Final soil water content</td>
<td>43.6</td>
<td>Potential soil evaporation</td>
<td>-273.6</td>
<td>-275.3</td>
<td>-298.1</td>
</tr>
<tr>
<td>Total irrigation water volume</td>
<td>20.7</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Total runoff</td>
<td>-4.5</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Total infiltration</td>
<td>16.1</td>
<td>Total infiltration</td>
<td>181.5</td>
<td>174.2</td>
<td>122.9</td>
</tr>
<tr>
<td>Precipitation + sprinkler irr.</td>
<td>32.8</td>
<td>Precipitation + sprinkler irr.</td>
<td>315.0</td>
<td>315.0</td>
<td>315.0</td>
</tr>
<tr>
<td>Total evaporation</td>
<td>-24.7</td>
<td>Actual soil evaporation</td>
<td>-236.8</td>
<td>-238.3</td>
<td>254.5</td>
</tr>
<tr>
<td>Total transpiration</td>
<td>-34.3</td>
<td>Actual transpiration</td>
<td>-340.5</td>
<td>-336.5</td>
<td>-295.4</td>
</tr>
<tr>
<td>Deep percolation</td>
<td>-0.6</td>
<td>Deep percolation</td>
<td>-5.9</td>
<td>-5.9</td>
<td>-5.9</td>
</tr>
<tr>
<td>Change in soil water storage</td>
<td>-10.0</td>
<td>Change in soil water storage</td>
<td>-78.8</td>
<td>-83.6</td>
<td>-111.3</td>
</tr>
</tbody>
</table>

Figure 75 presents the simulated soil water storage $S_{soil,s}$ in the full grown root zone (0.0 - 1.2 m depth) at $x_{inf} = 32.5$ m in comparison to the measured soil water storage $S_{soil,m}$ in the same zone at $x = 20$ m. $S_{soil,m}$ is a measure of the soil water volume stored in a soil column of a certain depth. For a given number of soil moisture measurements $m$ at different soil depths $z_i$, the soil water storage is calculated by

$$S_{soil,m} = 10^3 \left[ \sum_{i=2}^m \theta_i (z_i - z_{i-1}) + 1.5 \cdot \theta_1 z_1 + 0.5 \cdot \theta_m z_m \right]$$  \hspace{1cm} (180)

with $\theta_i =$ volumetric soil moisture content at the depths $z_i$.

The units of $\theta_i$ and $z_i$ are [m³/m³] and [m] respectively, which leads to the unit of $S_{soil,m}$: [10³m³/m²] = [mm]. The soil water storage calculated by Equation 180 is associated with the 1D soil column around the access tube for soil moisture measurements. In order to get a representative value of the cross section, $S_{soil}$ values corresponding to both the furrow and the ridge are averaged.
In contrast to $S_{\text{soil,m}}$, the FIM-simulated soil water storage of the root zone $S_{\text{soil,s}}$ is determined exactly by

$$S_{\text{soil,s}} = 10^3 \cdot \sum_{i=1}^{n} \frac{\theta_{e} \cdot A_{e}}{A_{\text{rootzone}}} \cdot \text{mm} \tag{181}$$

with

$$\theta_{e} = (\theta_{i} + \theta_{j} + \theta_{k})/3$$

$\theta_{i}, \theta_{j}, \theta_{k}$ = volumetric moisture content at three boundary nodes $i, j, k$, of the triangular element $e$ in the calculation mesh

$A_{e}$ = area of the element $e$ and

$A_{\text{rootzone}}$ = total root-zone area.

The soil water storage based on field measurements is subsequently calculated by Equation 180 whereas simulated soil water storage is computed by Equation 181.

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**Figure 75** Observed and simulated soil water storage at the upstream section $x_{\text{inf}} = 32.5 \text{ m}$ during the entire growing season (1999)

Figure 75 shows a very good agreement of the simulated soil water storage at $x_{\text{inf}} = 32.5 \text{ m}$ with the first field observations ($59^{\text{th}}, 61^{\text{st}}$ and $65^{\text{th}}$ day after sowing). This agreement can be considered as a positive confirmation of the chosen initial soil moisture distribution. The simulated increase of the storage due to both rainfall events and irrigation events ($45^{\text{th}}, 57^{\text{th}}$ and $91^{\text{st}}$ day after sowing) also compared favourably with the observations. Similarly, long drying periods ($73^{\text{rd}}$, $90^{\text{th}}$ day and $112^{\text{th}}$ - $138^{\text{th}}$ day) were correctly simulated. Thus, the complex and highly dynamic water flow towards, within and out of the root zone (due to infiltration, evaporation and transpiration) tallied well with the observations during the whole growing season.
Nothing can be stated about the short-time flow dynamics in the soil since high-resolution soil moisture data (in both space and time) were not available.

**Impact of irrigation practice on water storage**

In contrast to the FDF irrigation method, the CEF practice is characterized by a high downstream infiltration volume due to ponding conditions at the tail end section of the field. This is illustrated by the soil water storage of the Lavalette experiments, measured five days before harvest, which was $S_{soil,m} = 229.6$ mm at the upstream section ($x = 20$ m) and $S_{soil,m} = 329.5$ mm at the downstream section ($x = 110$ m). Figure 76 shows the observed soil water storage at the three measuring stations ($x = 10$ m, $x = 65$ m and $x = 110$ m). The storage was significantly higher at the downstream section as compared to both the upstream and the middle sections during the whole measuring period. The only small difference between the upstream and the middle sections indicates that the location $x = 65$ m was less affected by the backwater of the CEF practice.

On 21 July 1999 (56 days after sowing), $S_{soil,m}$ was already higher at the downstream section as compared to the other sections, which was probably a consequence of the first irrigation and infiltrated surface runoff from three heavy rainfall events. The downstream storage diverged (increased) significantly from the storage of the other sections in the time between the 3rd irrigation event and the 113th day after sowing. This was most likely also caused by heavy rainfall events with high surface runoff during this period (Figure 73) which led again to ponding conditions at the downstream part of the field.

The simulation of FDF practice by FIM resulted in a quite different soil water storage at the downstream part of the field as compared to the observations. Prior to the 3rd irrigation event for example, $S_{soil,m}$ was 217 mm at $x = 130$ m. In comparison, the soil water storage for an average rootzone matric head at field capacity ($h_{m}(FC) \sim -1.5$ m) was estimated to be about 260...280 mm. As seen in Figure 76, the $S_{soil,m}$ was above this threshold value at $x = 110$ m and it was significantly higher than the simulated value. It should be noted that the simulated

---

5 The volumetric water content at field capacity was computed from the soil hydraulic functions and varied in the three soil layers: $\theta_w(FC) \sim 0.26...0.28$ [m³/m³].
value matched well with the observed values of both the upstream and middle sections, which were not affected by the backwater effects of CEF.

The observations indicated water saturated soil at the tail end of the field between the 56th day after sowing and the harvest. On the other hand, the soil at both $x = 20$ m and $x = 65$ m was unsaturated for the most part of the growing period (Figure 76).

**Soil moisture profiles**

Volumetric soil moisture content $\theta_w$ was measured at various depths $z_l$. The function $\theta_w = f(z_l)$ is referred to as soil moisture profile. In order to make the FIM simulations comparable to the observations, soil moisture profiles were calculated by horizontal discretization of the subsurface flow domain according to the $z_l$ values and averaging $\theta_w$ within the resulting depth layers.

Figure 77 shows simulated and observed soil moisture profiles (at $x = 20$ m and $x_{inf} = 32.5$ m) at times before and after the irrigation events and at five days before harvest. Simulated soil moisture profiles compare favourably with the observations on the days before the irrigations (at 45, 57 and 90 days after sowing) as seen in Figure 77 a)-c). The observed moisture content immediately after the irrigations L2 and L3 showed a maximum at $z = 0.2$ m, whereas the simulated moisture in the upper 0.5 m soil depth was almost uniform\(^6\). The latter was due to the fact that HYDRUS-2D simulates the diffusive effect (matric flow) and does not account

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\(^6\) This was most probably also the case for the first irrigation, but measurements were available only three days after the event.
for possible preferential flow paths which may have existed in the field due to organic matter, remaining root channels, animal activity or soil cracking.

There was generally an excellent match of the observed and simulated moisture of the deeper soil layers. The observed moisture profile at the 133\textsuperscript{rd} day after sowing, however, differed from the almost stable simulated moisture as shown in Figure 77(d). In the upper soil layer (0.0 ... 0.6 m), the observed moisture content was higher, whereas it was lower in the subjacent layer (0.6 ... 1.2 m). These differences were almost counterbalancing in the integral water storage as seen in Figure 75. Steady soil moisture conditions were assumed at the 133\textsuperscript{rd} day after sowing because no significant rainfall events were observed prior to this date and the root water extraction was assumed negligible a few days before harvest. The distinct bent form of the observed soil moisture profile can have physical reasons: the porosity in the upper layer was probably higher than the presumed value (likewise $\theta_s$) and, similarly, the porosity of the layer 0.6 ... 0.8 m was probably lower than the presumed value. This again underlines both the difficulty and the importance of the estimation of realistic soil hydraulic parameters together with an accurate evaluation of distinct soil layers.

2D contour plots of simulated soil moisture distribution at $x_{inf} = 32.5$ are presented in Figure 78 for selected times. It should be noted that the furrow was not considered in the interpolation scheme which was developed to create the plots and used the nodal values of simulated moisture in the subsurface calculation mesh.

![Figure 78](image-url)

Figure 78 Simulated soil moisture distribution at $x_{inf} = 32.5$ m for various times during the Lavalette run

Figure 78a) shows a distinct moisture gradient at the soil surface due to a recent rainfall event (13.5 mm) and soil evaporation. After long drying periods without precipitation, the volumetric...
Irrigation control: towards a new solution of an old problem

water content was horizontally uniform and only changed with increasing soil depth as seen in Figure 78d). This was a consequence of the redistribution process but also of the assumed 1D development of the plant roots in the soil. This horizontal uniformity of $\theta_w$ existed below the root zone during the whole growing season. At the interface between the three different soil layers (Table 27), i.e. at $z = 0.55$ m and $z = 0.95$ m, a jump in moisture content was present throughout the entire simulation due to the different soil hydraulic properties of the individual layers (Figure 78a-d).

Figure 78b) and c) show the simulated moisture distribution at times before and after the 3rd irrigation event. The moisture distribution before the irrigation (Figure 78b) resulted from the relatively large water extraction rate from the root zone (0.0 ... 1.0 m depth at 90 days after sowing). There was a steep moisture gradient from deeper soil layers, where the water content was significantly higher, towards the lower boundary of the root zone. It was only after about 15 hours after irrigation that the simulated wetting front had advanced to a depth of 0.5 m (Figure 78c).

**Overall irrigation efficiency**

The irrigation efficiency IE is one frequently used criteria to evaluate the performance of an irrigation system during the entire growing season. In the present study, the 'beneficially used water' is equal to the amount of the crop evapotranspiration $ET_{c,a}$. The total irrigation efficiency is calculated in this context by

$$IE = \frac{ET_{c,a} - V_p - V_{CR} + \min[0, \Delta S_{\text{soil}}]}{V_{in}} \cdot 100\% = 79.1\%$$

Equation 182 takes into account that beneficially used water excludes water naturally supplied to the crop, e.g. by precipitation, capillary rise towards the root zone and water which is taken from the soil water storage $S_{\text{soil}}$. Water which adds to the soil water reservoir (root zone) is not counted and remains neutral until such time as it leaves the root zone [Burt et al., 1997].

This relatively high IE value must not be overrated since only about 27% of the water volume used for crop evapotranspiration was irrigation water. The larger share of the water was supplied by precipitation and sprinkler irrigation (cf. subsection 4.1.3.3). For the CEF practice, which was applied at the Lavalette run, the IE is in theory even higher than the calculated value because surface runoff is zero. Thus, 'non-beneficial' water can leave the flow domain (i.e. the field) only by deep percolation, which was about 1% of the total water balance in the case of the FDF simulation. By the CEF technique, however, the risk of deep percolation is higher at the downstream part of the field.

In this context it should be noted that deep percolation occurs if the water leaves the lower subsurface domain boundary at 2.0 m soil depth. Water which leaves the root zone (0.0 ... 1.2 m) towards deeper soil may move upward again in drier periods by capillary rise and is, therefore, not counted as a loss.

**Evapotranspiration**

The components of the simulated evapotranspiration, namely, soil evaporation and transpiration, are shown in Figure 79 for the duration of the entire growing season. The values are given in units of [mm] and associated with the cross section at $x_{\text{inf}} = 32.5$ m. The actual crop transpiration $TA$ was always close to its potential values $TP$. Along with an increasing crop transpiration, the soil evaporation decreased due to the shading of the soil by the growing
crop (the LAI increases). After crop maturity, the soil evaporation increased again because the leaves of the corn started drying up (and consequently LAI values decreased). The soil evaporation showed characteristic declining curves during dry periods (i.e. periods without precipitation) before the 1st irrigation event when the soil was exposed to direct radiation and it was still unprotected by the developing crop canopy (Figure 79). The decline was a consequence of the physical limit of capillary rise of soil water at a critical matric head $h_{crit}$ at the soil surface. This threshold value was set to $h_{crit} = -160$ m for the simulation of the Lavalette run by FIM. Soil evaporation ceased if the matric head at the soil surface approached $h_{crit}$. Values of both actual and potential soil evaporation were in unison at those times between the 50th day after sowing and the actual harvest. EA reached its characteristic minimum when the LAI was at its maximum (at around the 72nd day after sowing).

![Figure 79](image.png)

Figure 79  Simulated components of evapotranspiration at $x_{ref} = 32.5$ m during the entire growing season of the Lavalette run

**Leaf area index**

Figure 80 shows the observed LAI and the functions of both the simulated LAI and the potential leaf area index $LAI_{pot}$ at the cross section $x_{ref} = 32.5$ m. The latter was calculated by the crop model LAI-SIM under consideration of the observed atmospheric boundary conditions (air temperature T) and the assumption of optimal growth conditions, i.e. without plant water stress.

Eight measurements of leaf area index were conducted during the growing season at the upstream part of the field. Each LAI measurement represented the mean of individual samples. Although the number of samples was too small to make assumptions of normality...
distribution, the LAI values are plotted in Figure 80 together with the standard deviation $\sigma$ by admitting that 68% of the values were within the range of LAI $\pm \sigma$ [Mailhol, 2004a]. The simulated LAI perfectly matched the observations during the early growing stage until 56 days after sowing. Later in the season, it was higher than the observed values and reached its maximum on the 72nd day after sowing. After that, simulated LAI values were in decline, whereas the observations showed values of around 4.5 [m$^3$/m$^2$] on the 87th and the 99th day after sowing (Figure 80). These values were close to the maximum LAI value for the corn variety under consideration and by a plant density of 8 plants/m$^2$.

![Graph showing observed and predicted leaf area index LAI during the entire growing season of the Lavalette run](image)

**Figure 80**  Observed and predicted leaf area index LAI during the entire growing season of the Lavalette run

The observed LAI values indicated better growing conditions during the late maturity stage than during early maturity and, as a response, an increasing LAI. On the other hand, the observed soil water storage during late maturity was less than the storage during early maturity (cf. Figure 76) which does not support the above hypothesis. Moreover, it should be considered that LAI measurements are not easy to perform and may be prone to large errors. The accuracy and consistency of the corn-related measurements taken during the 1999 growing season are, however, not revisable.

**Yield**

The potential grain yield was calculated to $Y_m = 12.0$ t corn/ha by Equation 113 with the harvest index $HI = 0.5$ and the radiation use coefficient $RUE = 1.32$ g · MJ$^{-1}$. The adapted Equation 112 by Mailhol et al. [2004] was used for calculation of the actual grain yield $Y_a = 11.1$ t/ha. Taking into account that Equation 113 and Equation 112 are given for dry grain yield (i.e. a corn moisture content of about 0%) and an estimated grain moisture content of 15% at harvest, the calculated yield had to be multiplied by 1.15 in order to make simulation and observation comparable. The final result of simulated actual grain yield was $Y_a = 12.8$ t/ha, which matched perfectly with the observed average grain yield of 12.7 t/ha.
Irrigation strategies for satisfying crop water requirements concentrate on the avoidance of crop stress, which can be caused by extremely dry or extremely wet conditions. The area between both extremes defines a plant-specific corridor where the degree of soil moisture in the vicinity of the plant roots may vary. Moreover, the upper limit of acceptable soil moisture is limited by the field capacity of the soil. When the soil dries out and attains a prescribed certain lower limit of soil water content, irrigation takes place, i.e. the initial condition $\theta_{ini}$ of an irrigation cycle is generally given by measurements or simulations. The upper limit of admissible soil moisture can also be considered as the maximum filling of the soil water reservoir. Every single irrigation cycle aims towards the achievement of this state of soil moisture in the root zone along the field. Irrigation control as regards water application to the field can thus be defined by the following optimization problem:

$$Z_2 = \max_{q,t} \text{PAE}_{z=z_s}^l(q,t,\theta_{ini}) = \max_{q,t} \text{AE}(q,t,\theta_{ini}) \text{ mit } AD_{z=z_s}^l = 1 \quad (183)$$

The optimal solution $(q^*,t^*)$ with

$$\left(q^*,t^*\right) = \arg \max_{q,t} \text{PAE}_{z=z_s}^l(q,t,\theta_{ini}). \quad (184)$$

is represented by the optimal control parameters inflow $q^*$ and time of irrigation $t^*$. Being able to employ these parameters implies attaining the required soil moisture, with minimal water losses, in the root zone $z = z_s$ which is defined more precisely by $AD_{z=z_s}^l = 1$.

In addition to the condition $AD^l = 1$, the limits of the control parameters are defined by $t_{min}$, $t_{max}$, $q_{min}$, $q_{max}$. The values of $t_{min}$ and $t_{max}$ are generally predetermined by practical aspects such as, for example, the work load involved in preparing and executing the irrigation procedures and/or the availability of the irrigation. The maximum inflow $q_{max}$ is selected in such a way that erosion or furrow overflow is avoided. The smallest amount of irrigation water necessary for reaching the lower end of the field is the definition of the minimum inflow $q_{min}$.

Evaluating the objective function $Z_2$ necessitates a complete irrigation cycle including the redistribution of the irrigation water in the soil. The assessment of the irrigation process can be performed on this basis by comparing the irrigation efficiency of the water distribution with respect to the last quarter of the field. Obviously, this computation refers to a predetermined depth of the root zone $z_s$.

The procedure of optimal irrigation control using ANN can be illustrated by the means of a numerical experiment. This experiment demonstrates water application to a furrow with loamy soil which can be assumed exemplarily to be located in the middle of the field. The numerical model refers to a furrow with a length of 100 m, an upper width of 0.8 m and a soil depth of 2 m.

### 4.2.1 Furrow irrigation model

During the course of an irrigation cycle, hydrostatic pressure and soil moisture gradient are the driving forces behind the water transport processes, i.e. evaporation and transpiration only play a minor role during this relatively short period. Therefore, water movement only
concerns the coupled surface/subsurface flow processes. The model component FAP-H (see section 3.3.5.3) of the global irrigation model FIM describes surface flow on the basis of the zero inertia approach. The quasi 3-dimensional soil water transport is portrayed by a series of HYDRUS-2D models.

Numerical solution
The modelling of the aforementioned example furrow utilizes five HYDRUS-2D models along the furrow, together with the surface flow model (for the parameters, see Table 32). The resulting system of non-linear equations is solved numerically on the basis of a Newton procedure [Wöhling, 2005].

Table 32 Parameter of the surface flow model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>General parabolic profile</td>
<td>1.19</td>
<td>0.63</td>
<td>0.46</td>
</tr>
<tr>
<td>Length of furrow</td>
<td>100 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom slope</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strickler coefficient</td>
<td>20 m³ s⁻¹</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The computational mesh of the five HYDRUS-2D models consists of 1126 knots and 2131 triangular computational elements. Figure 81 shows the soil moisture distribution at the first three cross sections during an irrigation process.

Initial and boundary conditions
As far as the surface flow model is concerned, the upper boundary condition varies as shown in section 3.3.1 whereas the lower boundary condition is a priori unknown. This is also the case for the water table in the furrow, which represents the upper boundary condition of the subsurface flow model. Both are therefore computed during the simulation. A seepage face boundary condition represents the lower boundary condition of the soil water model. All other boundaries have a zero flux boundary condition.
Case studies: application of GAIN-P strategy

Soil water characteristics

The Mualem/van Genuchten model is used to describe the soil hydraulic parameters for our example (see Table 33).

Table 33  Soil hydraulic parameters of the loamy soil (taken from Roth [1996])

<table>
<thead>
<tr>
<th>α [m(^{-1})]</th>
<th>n</th>
<th>(θ_r)</th>
<th>(θ_s)</th>
<th>(K_s[10^{-5}\text{ms}^{-1}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>1.3</td>
<td>0.01</td>
<td>0.41</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Calculation of the water application efficiency

The computing of this type of irrigation efficiency is performed on the basis of the \(PAE^{lq}\) criteria 24 hours after the end of the irrigation event.

First of all this requires evaluating the maximum possible extension of the saturated root zone in the last quarter of the field, which still satisfies the conditions \(AD^{lq} = 1\). In this case, the irrigation per se is considered to have been satisfactory if the average soil moisture in the root zone is around field capacity. Subsequently, the water balance in the root zone of the entire field in relation to \(z_s\) and the initial starting time of the irrigation is computed and \(PAE^{lq}\) is evaluated.

4.2.2 Generating the training database

The furrow irrigation model together with the field-specific parameters are used for generating the data for training the ANN. The full range of the training scenarios is created by the systematic variation of the irrigation time, the inflow and the average initial soil moisture. Thus, the training databank is built upon the outcome of the repeated simulations of single irrigation cycles. The following goes into the methodology in more detail. The databank for training the ANN is generated by (i) variation of irrigation time \(t\), (ii) inflow \(q\) and (iii) averaged initial soil moisture \(\bar{θ}\) which all serve as input data for the irrigation model. A multitude of repeated simulations with these input values results in scenarios which produce, as the corresponding output, the maximum depth \(z_s\) as well as the irrigation efficiency criteria \(PAE^{lq}\) for each single scenario. All these essential parameters are assigned to the vector \((\bar{θ}_{ini}, t, q, z_s, PAE^{lq})\) and characterize an irrigation event. In more detail, the input parameters are selected from a uniform distribution as regards the intervals.

The input values are uniformly chosen from the following intervals:

\[q = [q_{min} = q_{min}(\bar{θ}) \leq q_{max} = 5 \text{ l/sec}], \quad \bar{θ} = [θ_{PW} = 0.11, \quad θ_{0.8FK} = 0.31] \quad \text{and} \quad t = [t_{min} = 0.25 \text{ h}, \quad t_{max} = 12 \text{ h}]\]. Preliminary experiments with the coupled surface/subsurface flow model provided, through linear regression, the relationship \(q_{min}(\bar{θ})\). This was necessary because average depths \(z_s\) often went hand in hand with high irrigation efficiencies for minimal inflow rates.

After having executed 10 000 simulations, the resulting 10 000 data vectors represent the provisional databank. This complete provisional training data \(D'\) is transformed into its final version which contains the single scenarios \(D\) with a frequency \(n \in \mathbb{N}\) in accordance with its efficiency criteria \(PAE^{lq}\).
A suitable frequency function $\eta$ was found to be

$$n = \text{floor} \left( \eta \left( \frac{\text{PAE}_{z=2}^{q} (\bar{\theta}_{\text{ini}}, t, q)}{10^{10}} \right) \right) = \text{floor} \left( 10^{2 \left( \frac{\text{PAE}_{z=2}^{q} (\bar{\theta}_{\text{ini}}, t, q)}{100} \right)} \right).$$

$$\text{(185)}$$

The maximum frequency of a training vector is limited to $n = 100$. Irrigation events with a poor efficiency are discarded from the training data if $n < 10^{0.7}$. The resulting final training data $D$ features, after transformation, 26,279 training vectors $(\bar{\theta}_{\text{ini}}, t, q, z_s)$.

### 4.2.3 Setting up and training of the SOM-MIO

To determine the optimal irrigation time $t^*$ and the optimal inflow rate $q^*$ we use a 2-dimensional SOM-MIO with ITRI option and hexagonal topology. It features four dimensional vectors $\tilde{m} = (\theta_{\text{ini}}, t, q, z_s)$. We now incorporate the selection matrix $D_x$ into the training step for finding the BMU. Taking $D_x = \text{diag}(1, 0, 0, 1)$ we ensure that the self-organizing map develops in the 2-dimensional plane defined by $\bar{\theta}_{\text{ini}}$ and $z_s$.

When applying this SOM-MIO for identifying the optional parameters $(t^*, q^*)$, the output layer still contains $D_x$ and the selection matrix for computing the output is $D_y = \text{diag}(0, 1, 1, 0)$. The training procedure of the SOM-MIO which contains 400 neurons starts after an arbitrary initialization of the four dimensional characteristic vectors. During the batch training, the learning rate changes with $\alpha_s = \frac{1}{1+k}$ in the course of 10 learning cycles. The neighbourhood radius $\sigma$ decreases from 10 down to 1.

### 4.2.4 Results: SOM-MIO as a tool for optimal control of water application parameters

The SOM-MIO has been developed as an alternative to the cumbersome and somewhat dubious classical approach to optimal irrigation control. In order to clearly illustrate the pioneering advantages of SOM-MIO over modern standard approaches we do not only assess the results of SOM-MIO just by test data, but also by the results of the traditional approach, i.e. nonlinear optimization procedures. The objective functions may have a multitude of local extrema. This characteristic necessitates relying not on optimization algorithms which are based upon derivatives of the objective function but on a global optimization algorithm. The latter recommends the *Shuffled-Complex-Evolution Algorithm* (SCE-UA) which combines the Simplex algorithm with evolutionary optimization techniques [Duan et al., 1993].

Figure 82 compares the results of SOM-MIO and SCE-UA on the basis of optimal irrigation parameters with respect to two fundamentally different initial conditions, i.e. an extremely dry field (Figures 82 a) and c)) and one with a moderate soil moisture deficit (Figures 82 b) and d)). Figures 82 a) and b) exhibit the optimal inflow and Figures 83 c) and d) the optimal

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7 $\text{floor}(x)$ rounds down to the next natural figure in the direction of zero.

8 $\text{diag}(d_{11} \ldots d_{nn})$ generates a diagonal matrix with the given elements.
irrigation time with respect to a given depth of the root zone, which has been supplied with irrigation water to satisfy $AD^q = 1$.

Figure 82 Calculation of the optimal irrigation parameters by SOM-MIO
The optimal irrigation parameters \( (t^*, q^*) \), evaluated by applying SOM-MIO, display a continuous graph in the above-mentioned Figures 82 a), b), c) and d). This high resolution is possible because the parameters can be evaluated extremely efficiently by SOM-MIO and this even after one single training process. In comparison, the SCE-UA algorithm requires a relatively long calculation time, i.e. almost 16 hours for completing one single optimization. This drawback seriously restricts the number of possible optimization runs with the SCE-UA algorithm and for this reason evaluations of SCE-UA can only be carried out on a random sample basis.

Despite the fact that the application of the SCE-UA to this task is a highly cumbersome procedure, the comparison of the results of SOM-MIO and SCE-UA shows a very good agreement and this is especially the case with regard to the optimal irrigation time.

**Very dry conditions**

The above picture distorts somewhat, however, with especially dry initial conditions, where the optimal inflow with high gradients leads to slight deviations between SOM-MIO and SCE-UA. This can be seen in Figure 82a, which also demonstrates a strong variation of the optimal inflow with respect to the root zone. Relatively shallow root zones require less irrigation water to be applied to the field. The irrigation, however, has to be executed with high discharge values because then irrigation is at its most efficient due to an improved uniform wetting. When the plant roots lie deeper, the necessary increase in the volume of required irrigation water is to be applied over a longer period of time, is however partly compensated for by a smaller inflow rate. These rule-of-thumb guidelines only apply for root zones down to a depth of 1 m. As far as more profound root zones are concerned with their additional water volume requirements, the available maximum irrigation time, as in our example, is not sufficient to allow distribution on the basis of the optimal small inflow rate. Here, the sub-optimal inflow increases proportionally with water volume and irrigation efficiency \( PAE_q \) decreases from 50% down to 30% within root zones located in 1 to 2 m depth.

**Moderate soil moisture deficit**

In less dry initial conditions, the maximum irrigation time \( t_{\text{max}} \) no longer represents a limiting factor as the required volume of water to be supplied to the root zone is substantially less. Thus, low inflow rates may also apply for root depths below 1 m (Figure 82b). Other than in dry conditions, a high inflow rate can be considered optimal down to a root depth of \( z_s = 0.5 \) m. Optimal irrigation time increases more or less linearly with an increase of the root depth, especially if the inflow rate remains approximately constant.

Figures 82 e) and f) show neurons which represent extremely dry (Figure 82e) and relatively moist (Figure 82f) soil moisture conditions. These figures convincingly illustrate the fact that neuron 1 represents the most shallow root depth and neuron 20 the deepest. In between, the development of the neurons exhibits a monotonic behaviour. This evidently well-balanced distribution of the neurons with respect to the root space is one reason for the excellent interpolation capacities of the SOM-MIO as regards the evaluation of the optimal irrigation parameters.

To compare the computation effort required for executing the SOM-MIO and the SCE-UA algorithms, we refer to a dual Pentium III (800 Mhz) PC. Execution time for the SCE-UA algorithm amounts to 16.24 hours on average for one single optimization, whereas the SOM-MIO needs only 1.7 seconds (!) for the same operation. Taking into account the fact that
establishing the SOM-MIO required 6.34 days, this leads to a break-even point after only 10 applications (!).

4.3 Scheduling of deficit irrigation systems using GAIN-P

The GAIN-P (the evolutionary algorithm with process modelling FIM) strategy for optimizing irrigation scheduling on the basis of the physically based irrigation model FIM can be performed along the lines of two different procedures:

i) Comparison of current approaches which optimize irrigation schedules on the basis of water balance modelling

ii) Comparisons using the most modern optimization procedures together with the furrow irrigation model FIM.

4.3.1 Comparison of GAIN-P with water balance approaches

The first type of comparison for the optimal evaluation of scheduling parameters is taken exemplarily from the water balance procedure proposed by Schütze et al. [2005a], which itself was derived from the optimization procedure described by Rao et al. [1988]. This employs dynamic programming procedures for optimizing the volume of irrigation water with respect to weekly irrigations. The modelling of water transport processes relies upon a discretized water balance model.

4.3.1.1 Methods

The subsequent investigation uses two well-known objective functions:

i) One objective function is based upon the sum of the weekly calculated transpiration $T(i)$ over the considered period (Equation 186)

$$\max \left( \sum T(i) \right)$$  (186)

ii) The other objective function relies upon the relative crop yield ($\text{REL}_Y$) expressed by the quotient of the current transpiration $T(i)$ and the potential transpiration $P(i)$, whereby a stress factor $SI$ features as an exponent (Equation 187):

$$\max \left( \prod \left( \frac{T(i)}{P(i)} \right)^{SI} \right)$$  (187)

Further incorporation of the stress factor $SI$ into the objective function allows taking into account the temporal variation of the water stress sensibility of the plants.

As regards the discretization of the irrigated water volume, two options have been investigated:

i) The use of a basic 2-step discretization, i.e. $0\text{m}^3$ and $5\text{m}^3$, only allows for the decision as to whether irrigation takes place or not. When the second case applies $5\text{m}^3$ are applied to the field ($\text{AET}_5$ and $\text{REL}_Y$ 5).
ii) Employing three discretization steps, 0 m$^3$, 5 m$^3$ and 10 m$^3$, permits, besides the decision whether to irrigate or not, the choice between two irrigation volumes (AET 5/10 und REL Y 5/10).

Due to the omnipresent limitations of computational resources, dynamic programming forbids employing optimization runs with a high resolved discretization of the irrigation water. Because water balance models do not allow for the evaluation – by model simulation – of the irrigation efficiency of single irrigation runs, the respective infiltration of the total available water volume had to be assumed.

In order to create a basis for a comparison between both approaches, the total volume of water applied to the field has to be compared with the one obtained by the new approach and multiplied by the values of irrigation efficiencies. These are on average 0.35 without optimal control of inflow (1) and 0.875 when the inflow is optimally controlled (2). Thus, the 100 m$^3$ used in the first instance (efficiency 0.35) as well as the 40 m$^3$ used in the second instance lead both to an effective infiltrated water volume of 35 m$^3$.

In addition to the crop yield for a water volume of 100 m$^3$ we also calculated the yields attainable for water volumes of 80 m$^3$ (effective infiltrated water volume of 28 m$^3$) and 120 m$^3$ (effective infiltrated water volume of 42 m$^3$). The calculations were based on a constant inflow of 1 litre per second (see Table 34).

Table 34 Comparison of the yield [t/ha] achieved by dynamic programming/water balance modelling with the one achieved by GAIN-P

<table>
<thead>
<tr>
<th>$I_{\text{max}}$</th>
<th>AET 5</th>
<th>AET 5/10</th>
<th>REL Y 5</th>
<th>REL Y 5/10</th>
<th>FIM mit $q_i = 1 l/s$</th>
<th>FIM mit $q_i = q_i^{\text{opt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9.29</td>
<td>-</td>
</tr>
<tr>
<td>35</td>
<td>6.39</td>
<td>7.19</td>
<td>7.15</td>
<td>7.15</td>
<td>10.82</td>
<td>10.57</td>
</tr>
<tr>
<td>40</td>
<td>7.54</td>
<td>7.65</td>
<td>7.95</td>
<td>7.21</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>42</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>11.28</td>
<td>-</td>
</tr>
<tr>
<td>45</td>
<td>8.73</td>
<td>8.84</td>
<td>9.42</td>
<td>8.88</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

4.3.1.2 Results: dynamic programming and water balance modelling versus the GAIN-P methodology

The crop yields achieved by dynamic programming are greatly inferior to the yields attainable by GAIN-P (see Table 34). The reason for the poor result of the dynamic programming lies in the necessarily strong discretization of the (underlying) water balance models. This makes it impossible to accurately simulate both the water transport processes in the field and the soil moisture content. Furthermore, the underlying water balance model is not in a position to take soil moisture distribution into account.

4.3.2 Comparison of GAIN-P with the most modern optimization procedures

In literature there are numerous examples of alternative optimization procedures for the optimization of general objective functions without constraints. Our optimization tool comprises Genetic Algorithms and Artificial INtelligence methods (GAIN). In order to pave the way for an objective comparison of these procedures with the GAIN-P optimization strategy, it is
necessary to provide the furrow irrigation model FIM for each of the compared approaches. In our comparative investigation we exemplarily show a contrasting application of GAIN-P, the well known Shuffle Complex Evolution (SCE-UA) and Simplex Annealing (SA) as well as the differential evolution (DE) approach. Due to the fact that none of the alternative optimization procedures is either able to directly take into account the required constraints or the variable dimensions of the search space, we need to base our comparison on a simplified task. This leads to an example with

i) a fixed number of nine irrigation events, and

ii) applying the reconstruction operator to each single schedule prior to an evaluation of the crop yield via FIM.

In an attempt to further reduce the complexity of the task for a low level comparison with the competitive technologies, the ensuing optimization problem comprises 18 optimization variables without constraints.

Further to the already outlined simplifications, the subsequent task additionally prescribes the irrigation times $t_{i=1,...,9}$ for the individual irrigation events.

Table 35  GAIN-P scheduling and results (irrigated water volumes) for comparing the various optimization procedures

<table>
<thead>
<tr>
<th>Day of irrigation</th>
<th>15</th>
<th>30</th>
<th>39</th>
<th>52</th>
<th>58</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>82</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irrigation volume[m³]</td>
<td>1.7</td>
<td>7</td>
<td>24.3</td>
<td>21.0</td>
<td>11.4</td>
<td>14.1</td>
<td>15.9</td>
<td>2</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The optimization runs were stopped after 1600 functional evaluations if the investigated optimization procedures had not already converged to a solution.

In Table 36 we see the crop yields achieved by the various procedures with both varied irrigation times and water volumes. In Table 37 we see the crop yields achieved on the basis of varied water volumes, yet with prescribed irrigation times. Except for the differential evolution in the case with the prescribed irrigation times, not one of the procedures succeeds in achieving the crop yield attainable by the GAIN-P algorithm. The relatively poor yields which were achieved even with the considerably simplified second task are, as seen in Figure 83, a consequence of the poor convergence behaviour of the other methods in the investigation. With the one exception of the proposed specialized irrigation optimization algorithm GAIN-P, which can ascertain for itself when, how often and how much to irrigate, not one (!) of the optimization procedures is capable of improving on the irrigation schedule which was prescribed at the outset. However, with respect to those investigated optimization procedures which are founded on furrow irrigation modelling (FIM), all of their respective crop yields were considerably superior to those attained with the help of dynamic programming on the basis of a discretized water balance model. The forecasting accuracy of the underlying irrigation model apparently plays a very important role in the optimization of irrigation schedules.

Table 36  Crop yields achieved by the various optimization procedures based on nine irrigation events with both different irrigation times and different volumes

<table>
<thead>
<tr>
<th>Method</th>
<th>SCE-UA</th>
<th>SA</th>
<th>DE</th>
<th>GAIN-P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield [t/ha]</td>
<td>10.57</td>
<td>10.15</td>
<td>10.02</td>
<td>10.82</td>
</tr>
</tbody>
</table>
Table 37  Crop yields achieved by the various optimization procedures based on nine irrigation events with varying volumes, yet with prescribed fixed irrigation times

<table>
<thead>
<tr>
<th>Method</th>
<th>SCE-UA</th>
<th>SA</th>
<th>DE</th>
<th>GAIN-P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield [t/ha]</td>
<td>10.23</td>
<td>9.84</td>
<td>10.61</td>
<td>10.82</td>
</tr>
</tbody>
</table>

Figure 83  Comparison of the convergence behaviour of different optimization algorithms

4.4  Application of GAIN-P to the simultaneous optimization of irrigation control and scheduling in furrow irrigation

Due to the difficulties discussed in the preceding chapters, the search for suitable optimization procedures for irrigation control and scheduling proves to be quite complicated, particularly as there is an additional problem to be added to the five already mentioned. For practical purposes, it is impossible to have a rapid convergence of the optimization procedure which, due to its complexity, unfortunately makes it impossible to solve the external optimization
A second possibility for reducing the computational effort lies in the acceleration of the inner optimization procedure by avoiding additional iterations. This can be achieved by using an ANN instead of a numerical optimization algorithm. With the help of an appropriate optimization algorithm, the ANN learns the optimal control for all the possible irrigation processes, and once training is completed, can be applied to the intersecting coupled optimization strategy.

The application of the coupled optimization strategy (GAIN-P) refers to a deficit irrigation in field experiments performed by Mailhol [2001] in Lavalette. For this experiment a field was used which was 130 m long and had a slope of 0.25%. Furrow irrigation was used; the furrows were 80 cm apart and 15 cm deep. The field consisted of silty loam and corn was chosen for the crop. Many investigations were carried out throughout the duration of the experiment (132 days) including suction head measurements, soil moisture and leaf area index measurements.

4.4.1 The furrow irrigation model

In an attempt to simulate different irrigation strategies, a strip of the field, with a furrow running down the middle, was set up in accordance with the already described dynamic furrow irrigation model FIM. Many calculations were carried out, which at the same time served to verify the model [Reuter, 2003]. The results of the simulations showed that FIM is able to give a realistic portrayal of the water transport processes and plant growth.

4.4.2 Solution of the external optimization problem

GAIN-P employs an evolutionary algorithm for solving the task-related external optimization problem. The theoretical implementation of the evolutionary algorithm to the problem of operational irrigation planning is shown in de Paly [2005]. In order to comply with the site constraints (Equations 12-14), the evolutionary algorithm was extended by a fourth operation known as reconstruction.

The newly developed evolutionary algorithm employs the following parameters for solving the envisaged task: population $n_{gon} = 20$, maximum number of evolutionary steps $n_{max} = 800$, the mutation rate for irrigation scheduling and water volumes $\sigma_d = 1, 2$ and $\sigma_V = 1.000$, respectively, the probability of recombination $p_m = 0, 33$ and the probability of crossover $p_C = 0, 5$. In order to remain within realistic irrigation practice, the irrigation planning generated by the evolutionary algorithm has to satisfy two conditions. Firstly, a minimal delay between two irrigation events of $d_{min} = 5$ days has to be guaranteed and secondly, the smallest irrigation water volume for a single irrigation event corresponds to $V_{min} = 1, 5$ m³. In order to illustrate the effect of an optimal irrigation control, the external optimization task is also solved without considering the optimal solutions of the internal optimization problem.
This leads to prescribing a constant inflow of \( q = 1 \) l/s, which was a reasonable value for dry as well as for relatively moist initial conditions.

### 4.4.3 Solution of the inner optimization problem

The optimal control of a furrow irrigation cycle employs the trained SOM-MIO. The functioning of the SOM-MIO within the interacting coupled optimization strategies is shown in Figure 84. During the simulation of a whole growth period using FIM, the SOM-MIO applies directly for each single irrigation event on the basis of the actual average soil moisture \( \bar{\theta}_i \), which is provided by the subsurface flow model and the water volume \( V_i \) which is provided by the evolutionary algorithm. With the help of these parameters, the SOM-MIO calculates the optimal inflow rate \( q^*_i \), which best meets the goal expressed by the objective function \( Z_1 \) (Equation 11), and thus controls the subsequent irrigation event which is simulated with the surface flow model FAP-H.

![Incorporation of SOM-MIO into the nested optimization strategy](image)

**Figure 84** Incorporation of SOM-MIO into the nested optimization strategy

#### 4.4.3.1 Generating the training and test data

The concept for generating the training and test data is extended by an optimization algorithm, which in our case is the Nelder-Mead-Simplex algorithm. For evaluating the optimal inflows, the input parameters are taken from an exponential distribution (log extension) from the intervals \( V = [1, 5 \, \text{m}^3, 20 \, \text{m}^3] \) and \( \bar{\theta} = [0.05, 0.35] \).

Subsequently, the Nelder-Mead-Simplex algorithm, using the sub module FAP-H, evaluates the optimal inflow \( q \) which maximizes \( AE^{q_i} \). The thus generated vector \( (V, \bar{\theta}, q) \) is then standardized and added to the training data \( D \) and test data \( T \). This procedure is executed 4000 times until \( D \) and \( T \) both contain 2000 data vectors each.

#### 4.4.3.2 The set-up and training of the SOM-MIO

A two-dimensional SOM-MIO with ITRI extension is chosen for learning the optimization problem. Pre-investigations with two SOM-MIOs featuring 100 and 121 knots respectively were carried out in order to find the required number of neurons. As there was practically no difference in the training error of the two SOM-MIOs, we did not try out any more
configurations and just used the SOM-MIO with 121 knots for further investigations. Before the learning process began, we undertook a random initialization of the three dimensional characteristic vectors. We used a batch-learning algorithm characterized by a receding learning rate \( \alpha = \frac{0.05}{1+k} \), whereby the neighbourhood radius \( \sigma \) decreases in the same period from 5 down to 1.

4.4.4 Case study: application of the GAIN-P strategy to the irrigation planning and control of a corn crop

The case study for operational irrigation planning and control of a corn crop should be able to clarify three questions:

i) How well can the SOM-MIO solve the inner optimization problem?

ii) How does the application of SOM-MIO speed up the evolutionary algorithm for solving the external optimization problem?

iii) What are the advantages of a simultaneous optimization of both irrigation planning AND control, when compared with current practice which focuses on irrigation planning only?

The vectors of the test data and the characteristic vectors of the SOM-MIO neurons are shown in a diagram in Figure 85a). It is plain to see that the characteristic vectors give a very accurate portrayal of the test data. This is confirmed by a very small mean squared error of \( mse = 0.002 \). When analyzing the results of the optimization runs it becomes clear that, if only small water volumes are available, a bigger flow rate is preferable and vice versa.

![Diagram](image)

(a) optimal inflows (test data and SOM-MIO results) - standardized  
(b) histogram of irrigation efficiencies with and without optimal control

Figure 85 Training results of the SOM-MIO (a) and results of the nested optimization (b)

After successful testing, the SOM-MIO was built into the interacting coupled optimization strategy together with the developed EA and the FIM irrigation model. For illustrating the

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9 This setup is referred to as EA-SOM-MIO.
impact of the current practice on irrigation efficiency, where the focus is solely on irrigation planning, a second experiment was set up with a constant flow rate of 1 l/s.\footnote{This setup is referred to as EA-Q0.}

For the deficit irrigation with EA-Q0, we had a total of 100 m³ of water for the entire growth period. With this amount we were able to achieve a corn yield of 10.49 t/ha but with an efficiency of only 35%. In compliance with the EA-SOM-MIO computations, the $V_{pes}$ had to be reduced down to 40 m³ because, thanks to the optimal control, an efficiency of 85% was achieved and thus the term 'deficit irrigation' was no longer applicable. With the assistance of the EA-SOM-MIO's optimal irrigation planning and control we were able to achieve a similarly high corn yield of 10.49 t/ha but, this time, by using only 40% of the water volume which was available at the outset, i.e 60% of the initial water volume (!) was still available for further crop supply.

Based on the evaluations of the objective functions for EA-SOM-MIO and EA-Q0, Figure 85 b) shows the histogram with respect to the efficiency of an optimization run of the EA. It is plain to see that optimizing the water distribution achieves a far greater efficiency than is the case with a general flat rate stipulation of a constant flow. One of the reasons for this large discrepancy can be a poor initial estimate for the flow rate. A more realistic initial estimate would, however, only lead to a slight improvement in the efficiency throughout the growing period because for individual irrigation events the irrigation schedule dictates different volumes of water at different times. As can be seen in Figure 85 a), particularly for dry initial conditions, the given water volume has a great influence on the optimal flow rate, i.e. the optimal flow rate varies greatly between the individual irrigation events. Besides the SOM-MIO's improvement of the irrigation efficiency, we also looked at its influence on the convergence and computational efficiency of the EA, with respect to solving the external optimization problem. Here we need to mention two positive effects of the SOM-MIO which, independently of each other, lead to a considerable reduction in the EA's computation time. The first positive effect is the shortening of the computation time by directly solving the inner optimization problem. The Simplex algorithm can be seen as an alternative to the SOM-MIO; it was used to generate the training data and required on average 12 iterations before finding the solution. If one disregards the learning effort required, this means that the EA's computational time is reduced to a 17th (!) of the original duration and that, for the first time, it becomes thoroughly possible to make such applications of the evolutionary algorithm with a standard PC.

The second positive effect of the SOM-MIO is an enormous speeding up of the EA's convergence when compared with the time needed for this in the optimization run with constant flow rate. This effect was investigated in a second optimization run, whereby the stopping criteria of a maximum number of evolution steps was altered to a desired accuracy of $\epsilon_s = 0.1$ (see Figure 86b). On the grounds of optimal irrigation control, the EA-SOM-MIO model finds the convergence seven times faster (!) than the EA model with its constant flow rate. Using a standard PC (2GHz) the EA-SOM-MIO model required a computational time of only approximately 6 hours.
4.5 Application of the SOM-MIO to an irrigation experiment

The new methodology was applied to a trickle irrigation experiment which was performed by Meshkat et al. [1999] for modelling subsurface flow as well as for solving different inverse problems. They investigated the efficiency of the sand tube irrigation (STI) method and validated their numerical model using a laboratory experiment (Figure 87). An axisymmetric version of the SWMS2D model, a former freeware version of the Hydrus-2D model, was applied for simulating a quasi three-dimensional flow pattern.

4.5.1 Irrigation experiment

The irrigation experiment was conducted on a cylindrical lysimeter (diameter =1 m, height = 0.7 m, see Figure 87) filled with a Maury silt loam. A sand tube, located in the upper part of the domain, had a 9 cm radius and a depth of 28 cm. The diameter of the sand tube was chosen on the basis of the extent of water spread over the soil surface during the applied drip irrigation treatment. The sand tube was dimensioned such that the capillary rise of water in the soil matrix would be less than the total height of the sand column.

The database for training the SOM-MIO was generated using the same numerical model and identical model parameters as described by Meshkat et al. [1999]. Thus, the van Genuchten parameters for the sand used as inputs to the model are \( \theta_r = 0.02, \theta_s = 0.42, \alpha_{VGN} = 0.024 \text{cm}^{-1}, n_{VGN} = 4.13, K_s = 180.0 \text{ cm/h} \) and for the Maury silt loam \( \theta_r = 0.14, \theta_s = 0.525, \alpha_{VGN} = 0.024 \text{cm}^{-1}, n_{VGN} = 1.393, K_s = 1.8 \text{ cm/h} \). We also followed Meshkat et al. [1999] in implementing an impervious barrier (assumed \( K_s = 0.000018 \text{ cm/h} \)) along apart of the vertical sand column.

Figure 86  Comparison of the EA’s results with and without optimal control of the water distribution
Irrigation control: towards a new solution of an old problem

4.5.1.1 Initial and boundary condition
A constant hydraulic head was used to initialize the computation. The irrigation process was characterized by a constant flux boundary with a constant application rate (variations between 0.6 l/h and 6 l/h) into the sand tube (between grid points O and H, Figure 87). In the domain of the silt loam (between grid points H and D), the upper boundary condition was described by a potential evaporation rate of 0.05 l cm/h. All other boundaries had a zero-flux condition.

4.5.1.2 Finite element grid
The computational grid used for the numerical simulations differs from the one chosen by Meshkat et al. [1999]. Whilst they applied a quadrilateral grid (Figure 87) with 1376 nodes and 1302 elements, we generated a triangular grid of 1282 nodes and 2443 elements with MESHGEN, a supplementary tool of the Hydrus-2D 2.0 software. However, the number of grid-points used in the actual simulation was comparable to the one used by Meshkat et al. [1999] since any quadrilateral grid is subdivided into triangles by the numerical flow model before performing the computation. Thus, the simulations, shown in Figure 88, resulted in soil moisture patterns comparable to those presented by Meshkat et al. [1999].

4.5.2 Set-up of the SOM-MIO

4.5.2.1 Generation of the database
For this application, the SOM-MIO was designed in a three-dimensional structure. The computation of the subsurface flow was carried out for a simulation time of 12 h. The varying initial pressure head $h_{ini}$ at the upper boundary was the first variable in the permutation scheme for generating the set of feature vectors $(h_{ini}, t, q_x, z_x, z_u)$ which form the training
Case studies: application of GAIN-P strategy

The other components of the feature vectors are the application time \( t \), the application rate for irrigation \( q_a \) and the wetted depths \( z_{sx} \) and \( z_{sz} \), defined by \( 0.95 \theta_s \) (see Figure 88). The scenarios were created using initial conditions \( h_{ini} = \{-1000 + i \ast 20cm\}_{i=0}^{45} \) and an upper boundary condition \( q_a = \{0 + j \ast 0.606 l/h\}_{j=0}^{10} \). This resulted in a total of 460 simulations. The decision variable was defined as the depth of the wetting front in the z-direction \( z_{sz} \) and in the x-direction \( z_{sx} \) at a given time \( t = \{0 + k \ast 864s\}_{k=0}^{50} \). This procedure created a database consisting of 460 * 51 = 23460 feature vectors \( (h_{ini}, t, q_a, z_{sx}, z_{sz}) \). Before training the SOM-MIO, the calculated feature vectors were separated into a 'training' data set and a 'test' data set, each consisting of 11730 feature vectors. The test data were used solely for evaluating the accuracy of the SOM-MIO in solving the inverse problem. All components were equally weighted by normalizing the features \( x_{norm} = \frac{x_i - \min(X)}{\max(X) - \min(X)} \).

4.5.2.2 Training the SOM-MIO

The three-dimensional SOM-MIO, consisting of 2000 neurons, was trained using learning parameters proposed by Kohonen [2001]. The initial learning rate was set at \( \alpha_0(0) = 1 \). The neighbourhood radius around the BMU was set to decrease from initially \( \sigma(0) = 3 \) to 1 during subsequent training steps. The training was initiated by presenting all the 11730 feature vectors \( (h_{ini}, t, q_a, z_{sx}, z_{sz}) \) of the training data set to the SOM-MIO. With an increasing number of repeated presentations of this complete training data, the approximation of the
training data achieved a satisfactory result already after 987 sec on a Dual Pentium I (800 Mhz) PC.

4.5.3 Application of the trained SOM-MIO

The trained SOM-MIO was employed to perform four different tasks, i.e. (i) deterministic subsurface flow modelling, (ii) stochastic subsurface flow modelling, (iii) deterministic inverse modelling, and (iv) stochastic inverse modelling. For the stochastic modelling tasks a Monte Carlo technique was used to investigate the effects arising from the uncertainty and variation regarding the application rate $q_a$ of the applied drip irrigation equipment. A normally distributed random number generator $f_n$ with a mean of zero and standard deviation $\sigma_q = 0.3$ provided the stochastic component of the fluctuating application rate $q_a^*$ in each Monte Carlo realization

$$q_a^* = q_a + f_n(x | \mu, \sigma_q).$$ (188)

4.5.3.1 Deterministic subsurface flow modelling

The first application of the trained SOM-MIO consisted of a subsurface flow simulation for evaluating the vertical and horizontal saturated depth for a given initial hydraulic condition, the water application time and a certain water application rate $(h_{ini}, t, q_a \rightarrow z_{sx}, z_{sz})$. The water flow in the soil monolith (Figure 87) predicted by the SOM-MIO was consistent with the simulated water flow of the Hydrus-2D model (Figure 91a) and thus also with the transition of the wetted depth as observed by Meshkat et al. [1999].

4.5.3.2 Stochastic subsurface flow modelling

Monte Carlo subsurface flow simulations were carried out with the numerical model and the SOM-MIO for an exemplary application rate $q_a = 3 \text{ l/h}$ and an initial pressure head $h_{ini} = -800 \text{ cm}$. The number of simulations $n_{mc}$ which are required statistically in order to get representative first and second moments of the output $z_{sz}$ was determined by comparing the results of the deterministic scenario and the mean of the outputs from an increasing number of Monte Carlo runs at 50 selected times. The necessary number of Monte Carlo runs was assigned to $n_{mc} = 750$ where the mean value and second moments of both were essentially identical (see Figure 90).

Figure 91 a) shows the spectra of the scattered values of the wetted depth in $z$-direction for different irrigation times $t$. The variability of $z_{sz}$ due to deviations in the application rate increases significantly with irrigation time. Furthermore, it is worth noting that the type of the distribution changes with time to an asymmetrical probability distribution. Figure 91 b) shows the 5th to 95th percentile of the distribution of $z_{sz}$ with time. In this example the agreement between the two Monte Carlo simulations is very good. The similarity between the results from the Hydrus-2D model and the SOM-MIO indicates that the SOM-MIO can be employed in Monte Carlo studies without further training. This also enabled us to use the trained SOM-MIO for the subsequent inverse Monte Carlo study and to avoid additional simulations with the Hydrus-2D model.
4.5.3.3 Deterministic inverse modelling

The subsequent applications refer to solving the inverse problem, i.e. evaluating the water application time for a specified initial soil moisture (hydraulic head), an application rate and a desired horizontal extension \( (h_{ini}, q_a, z_{sx} \rightarrow t) \) (Figure 89a) or the vertical extension of saturation \( (h_{ini}, q_a, z_{sz} \rightarrow t) \). Moreover, the trained SOM-MIO was applied for calculating the application rate corresponding to given depths of saturation, initial soil moisture and application time \( (h_{ini}, t, z_{sx} \rightarrow q_a) \) or \( (h_{ini}, t, z_{sz} \rightarrow q_a) \) as shown in Figure 89 b). The comparison between the numerical simulation and the outcome of the SOM-MIO was based on the test data which was ‘unknown’ to the neural network, i.e. not used in the course of the training. The solution of the inverse problem requires evaluating the water application time or the application rate for a specific initial condition and a desired extension of the saturated region. Figure 89 a) shows the irrigation time necessary to obtain a certain desired horizontal or vertical saturation. The results are directly comparable to the water application times determined by the numerical simulation. Comparing the outcome of the SOM-MIO with the test data provided highly satisfactory results also with respect to the water application rate corresponding to a specified saturated depth (Figure 89b). Similar results were achieved for scenarios with different initial conditions, varying from soils near saturation to soils with very dry initial conditions \( (h_{ini} = -1000 \text{ cm}) \).

4.5.3.4 Stochastic inverse modelling

The Monte Carlo inverse calculation was performed for the inverse problem \( (h_{ini}, q_a, z_{sx} \rightarrow t) \), i.e. for evaluating the water application time for a specified initial hydraulic head \( h_{ini} = -800 \text{ cm} \), a mean application rate \( q_a = 3 \text{ l/h} \) and a number of 8 desired vertical saturated depths. The Hydrus-2D model was used in a combination with the Nelder-Mead-Simplex algorithm in
order to solve the chosen inverse problem for each given \( z_{sz} \) in each Monte Carlo realization. The number of vertical extensions was restricted for two reasons. Firstly, the Monte Carlo inverse calculation with the Hydrus-2D model required extensive computation time (around 14 days for this application) and secondly, the trained SOM-MIO did not cover the whole range of spectra data below \( z_{sz} = 20 \text{ cm} \).

Figure 92 a) shows the Monte Carlo spectra produced by the SOM-MIO. The variability of the irrigation time increases dramatically with the wetted depth. Similar to the stochastic subsurface flow modelling, the normally distributed application rate is transformed into a
positively skewed distribution. Comparing the relative variation in the two stochastic applications (Figures 91 a) and 92 a)) shows that the prediction of the irrigation time is more uncertain. Figure 92 b) displays five different percentiles (the 5\textsuperscript{th}, 25\textsuperscript{th}, median, 75\textsuperscript{th} and 95\textsuperscript{th}) of the predicted irrigation time with wetted depth from the numerical and neural framework, respectively. Again a good agreement between the results of the two inverse Monte Carlo techniques is observed.

4.5.3.5 Performance analysis

We investigated the computational efficiency of the Hydrus-2D model and the SOM-MIO on a Dual Pentium III (800 MHz) PC. Figure 93 shows the CPU time of the SOM-MIO required for generating the training data, training and application for separately solving all of the four above tasks. In addition, the computational effort of the numerical framework with an increasing number of applications is presented. In each graph the break-even point is given which indicates when it becomes worthwhile handling one of the pre-mentioned problems with an ANN, namely, with the SOM-MIO.

Figure 93 a) clearly shows the differences of the computational performance with an increasing number of applications for both strategies. Since the computation time of the numerical model is constant on average for each simulation, the neural approach incurs a relatively high initial effort for training. This becomes less cumbersome when dealing with the inverse problem (Figure 93 c) and becomes even less significant in the case of a Monte Carlo simulation (Figure 93 b) and d). Both subsurface flow modelling and the inverse calculation in a Monte Carlo setting benefit from the very first application from the ANN-based strategy. The new methodology offers yet another improvement in the performance when one considers that the new SOM-MIO is able to perform the different tasks after only one single training.
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The speed, robustness and stability of ANN-based applications could prove to be useful when dealing with Monte Carlo methods or optimization problems in water resources, where a large number of realizations of a numerical model are required for obtaining an appropriate solution. A new ANN architecture based on self-organizing maps was developed, which already after one single training, allows simulation tasks to be performed as well as inverse problems to be solved: the Self-Organizing Map with Multiple Input/Output option (SOM-MIO). This new architecture combines the superior clustering capability of the SOM with a linear interpolation scheme in order to generate continuous output information.

The SOM-MIO was comprehensively analyzed in order to explore its capabilities for both simulating soil water transport and solving an inverse problem of the Richards equation. The sensitivity analysis focuses primarily on the way(s) in which the network size and the number of sample vectors used in the training affect the performance of the SOM-MIO. The sensitivity analysis shows that the accuracy of the SOM-MIO generally depends on the size of the network. In addition, the SOM-MIO already achieved a highly satisfactory level in prediction accuracy with a relatively moderate amount of training data. However, the performance of the trained SOM-MIO providing three different types of mapping functions revealed some remarkable deficiencies regarding the nonlinearity and the border regions of the sample space.

In a second investigation, a number of extensions related to the improvement of the performance of the basic SOM-MIO were examined. A trial and error search could identify a combination with an excellent performance in predicting soil moisture transfer as well as in

Figure 93
Computational complexity (CPU time in sec) of the classical framework (dashed lines) and the ANN-based strategy (solid lines) with an increasing number of applications. Black labels indicate the break even point between both strategies.
finding the solutions of the inverse problems. A Delaunay interpolation scheme together with an enhancement of the training data at the borders and the nonlinear regions of the sample space proved in this analysis its outstanding ability to represent several related mappings coexisting in a single and coherent SOM-MIO.

Thirdly, these facts were demonstrated by the means of a more realistic and, in terms of dimensions, a more sophisticated example from irrigation practice, namely, a quasi three-dimensional numerical analysis of a trickle irrigation published together with a laboratory experiment. Comparing the results of SOM-MIO with the outcome of the numerical and laboratory experiments demonstrated the excellent performance of the SOM-MIO with respect to accurately simulating the 3D subsurface flow as well as its reliability in solving different inverse problems.

Besides its excellent prediction accuracy, the unconditional robustness and high computational efficiency of the SOM-MIO along with its ability to deal with both the simulation as well as the inverse task all recommend this new tool for a wide range of promising applications in water resource problems especially as regards optimizing irrigation efficiency.
Summary and future trends

Water is a limited resource and the dramatically increasing world population requires a significant increase in food production. The enormous challenge of feeding an additional two billion people in 2030 can only be met by an expansion in irrigated agriculture. For this reason, the FAO calls for a revolution in irrigation water management in order to improve the generally poor water-use efficiency in irrigation. Current practice aims at optimizing both crop yield and water use efficiency; however, the available tools have serious drawbacks.

- The application of physically based flow models for adequately describing the water transport processes necessitates numerical solution procedures. Their operation, which requires a certain level of numerical expertise, is too complicated for general routine use in irrigation practice. Therefore, irrigation practice generally replaces these models either fully or in part (e.g. as with subsurface flow modelling) by rough empirical models. Unfortunately, these approaches are more or less restricted by the range of their calibration data provided by a necessarily limited amount of field experiments.

- Potential problems with numerical instabilities and enormous CPU time requirements prohibit the use of rigorous process models and non-simplified approaches for optimizing water use efficiency on the basis of classical optimization strategies; this is even the case at irrigation research centres with experienced staff. Instead, a rough consideration/estimate of the water balance tends to form the basis of current tools for optimizing irrigation efficiency.

- Optimizing the irrigation parameters over a complete growing season, even on the basis of simplistic empirical flow models, already leads to a nonlinear optimization problem which represents an extremely complex task. Classical approaches for its complete solution require extremely cumbersome and highly sophisticated procedures which render the task practically unfeasible.

- The complex optimization of both irrigation control and scheduling for improving irrigation efficiency is commonly performed in separate procedures. Both are, however, closely interrelated and, consequently, a simultaneous optimization is really what is required for finding the best combination of the irrigation parameters.

These abovementioned shortcomings of the existing approaches are characteristic of the following, apparently contradictory, needs, namely:

- On the one hand, an improved irrigation efficiency requires tools for reliably determining the optimal irrigation management strategy for an envisaged crop pattern. This can only be achieved by a simultaneous optimization of both irrigation control and scheduling, together with sound process-based irrigation models which are able to consider a whole growing season.

- On the other hand, irrigation practice also needs simple, robust and efficient tools with rather modest computational requirements for the straightforward optimization of water
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application efficiency with respect to irrigation control and scheduling for a certain crop. Sophisticated procedures which require numerical expertise are not suited for practical applications.

This contribution demonstrates that a way out of this dilemma can be found by leaving the beaten track of classical modelling analysis. Instead of executing the cumbersome computations of all the irrigation-related, interacting water transport processes when solving a certain optimization task, the proposed philosophy alternatively performs the lion's share of the computational effort during a first preparatory phase prior to the solution of the optimization problem (Figure 94). During the preparatory phase (Figure 94), it thus offers a trouble-free yet complete and reliable solution to the optimization problem under consideration. The innovative GAIN-P methodology provides the tools necessary for the implementation of this alternative approach. It combines Genetic Algorithms, Artificial INelligence techniques and rigorous Process modelling for resolving the conflict.

As the preceding development and comparative analysis demonstrated, the new strategy paves the way for:

- a non-complicated and efficient simultaneous optimization of both the water application parameters (irrigation control) and the scheduling parameters over a whole growing season;
- a substantially improved prediction reliability by the option of basing the solution upon sound, physically based irrigation models which consider all the irrigation-relevant processes;
- a dramatic reduction of the computational effort because the remaining, relatively modest calculations for applying GAIN-P to the effective optimization task are straightforward and can be executed easily without numerical expertise.

The GAIN-P methodology performs the evaluation of the most beneficial (optimal) irrigation parameters in two fully separate phases, namely, a preparatory phase and an application phase.

The steps of the preparatory phase (Figure 94) consist of the following:

- when dealing with furrow irrigation: setting up the comprehensive physically based irrigation model with an adequate crop growth model coupled to a process-oriented surface-subsurface model on the basis of the data supplied by the investigated irrigated area (see Figure 94).
- when evaluating optimal water application for trickle irrigation systems: the surface-subsurface furrow flow model is simply replaced by a quasi 3-dimensional axisymmetric subsurface flow model which forms the tool for analogously generating the training database.
- using the physically based irrigation model for simulating a large number of irrigation scenarios over a whole growing season and for a selected crop type. They cover the whole range of all realistically feasible combinations of initial soil moisture prior to irrigation as well as water application and irrigation scheduling parameters. The totality of the resulting input/output relationships, expressed by all the different input parameters together with the corresponding output parameters, finally forms the database for subsequently training the task-specific ANN.
- teaching the problem-adapted artificial neural network with the training database until it fully and reliably portrays the input/output relationship originally provided by the complete irrigation model.
Figure 94  The global optimization problem in furrow irrigation
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The second phase, the application phase (Figure 94), consists of simply performing the remaining computations with respect to the actual task, i.e. solving the originally complex optimization problem by the relatively easy and straightforward application of GAIN-P, which was previously set up for the considered irrigation system. This provides the crop-optimal water application parameters for a given initial soil moisture and a desired soil water distribution within a tailor-made evolutionary optimization technique. Thus, the proposed approach accounts for both variable irrigation intervals and variable irrigation parameters throughout the complete growing season. The new strategy can be applied to both fully irrigated systems and to those with deficit water supply.

To execute the steps of the preparatory and application phases, GAIN-P builds upon three sub-modules:

- The process-oriented and physically based furrow irrigation model FIM essentially consists of three interacting sub-modules: a 1D surface flow model, a 2D subsurface flow model and the crop model LAI-SIM. The semi-analytical zero-inertia surface flow model reliably describes the discontinuous surface flow (e.g. advancing wave), whereas the numerical HYDRUS-2D model portrays the subsurface flow phenomena during infiltration and redistribution, thus providing a prognostic insight into the interacting surface/subsurface flow processes. The crop model evaluates the root growth, performs the leaf area simulation and indicates a potential plant water stress by considering the climatic conditions such as evaporation, rainfall, etc. Thus, FIM is in a position to yield the input/output vectors for a selected crop pattern obtained from all the realistically meaningful irrigation scenarios which finally generate the training database.

- The SOM-MIO, an artificial neural network designed with respect to the considered task. It represents a further development of the Self-Organizing Map (SOM) architecture. Using the aforementioned database for the training, the SOM-MIO straightforwardly provides, for example, the optimal flow rate and cut-off time with respect to the given water volume for a certain initial soil moisture distribution along the furrow. This internal optimization of the water application parameters is directly connected to the external optimization of the scheduling parameters.

- The problem-adapted Evolutionary Algorithm. It is coupled iteratively with the internal optimization and, thus, it finally accounts for the global nested optimization procedure by simultaneously evaluating the irrigation schedule together with the water application parameters proposed by the SOM-MIO. The global optimization of irrigation control and scheduling parameters considers the whole growth period and finally leads to the optimal global solution.

Alternatively, the GAIN-P methodology can also be applied to optimize water application in trickle irrigation systems. In this case, the surface/subsurface flow model is replaced by a quasi 3-dimensional axisymmetric HYDRUS-2D model which then, together with the other sub-modules, forms the tool for analogously generating the training database.

The new optimization strategy and each of the sub-modules were thoroughly examined, analyzed and compared objectively to the current modelling and optimization techniques. GAIN-P was first applied to furrow irrigation, tackling the difficult subject of a more effective deficit irrigation, i.e. optimizing water application and scheduling parameters for a complete growing season in order to obtain maximum yield from a limited volume of water. To achieve this goal reliably, a physically based hydrodynamic irrigation model was set up and iteratively coupled with the 2D subsurface flow model HYDRUS-2D on the basis of the local field data of the considered irrigated area. Portraying the local growth characteristics by
the crop model LAI-SIM by additionally taking into account the crop type, together with the climatic conditions such as evaporation, rainfall characteristics, etc. completed the furrow irrigation model of the site. The coupled irrigation model FIM served to generate the training database for all realistically feasible scenarios of water applications with respect to the furrow irrigation of a corn field.

The calibrated and validated process model was then used for training a problem-adapted artificial neural network, the SOM-MIO. A sensitivity and error analysis of the novel SOM-MIO network architecture demonstrated not only its impressive prediction reliability but also its uniqueness with respect to its ability to accurately and straightforwardly perform both the simulation task and the (otherwise highly cumbersome) inverse solution of the hydrodynamic furrow irrigation model within the same network. This means that the SOM-MIO has to be trained only once to execute both tasks, a fact which enormously speeds up the overall performance of the complete optimization tool. A subsequent investigation of the genetic (evolutionary) algorithm with respect to solving the nested optimization problem illustrated that the innovative GAIN-P methodology reliably finds the overall optimal combination of irrigation control and scheduling parameters to provide the maximum crop yield for a given water volume. In order to assess the results of a specific optimization scenario, a comprehensive sensitivity analysis was performed on the basis of the coupled furrow irrigation model FIM. By simulating soil evaporation, precipitation, root water uptake by the plants, as well as crop growth and yield over the whole growing season, we were able to disclose the impact of different irrigation schedules and water application parameters on crop yield. The results confirmed the convincing reliability of the proposed strategy. A comparative analysis with the current SCE-UA algorithm and with dynamic programming on the basis of water balance considerations was performed for furrow irrigation during a growth period of corn. The result demonstrated a striking superiority of the new strategy with respect to the achieved irrigation efficiency.

The reliability and convincing computational efficiency of the new strategy were attested by the results of a comparison with a second example in irrigation practice (taken from a recent publication), namely, a quasi three-dimensional numerical and experimental analysis of trickle irrigation. The axisymmetric version of the HYDRUS-2D model provided the training database of the SOM-MIO which was then applied to simulate the horizontal and vertical extension of the wetted soil volume as a consequence of both a given irrigation time and an initial soil moisture profile. A comparison was carried out between the GAIN-P results and those given in the article for the outcome of the numerical and experimental analysis. The comparison demonstrated the far superior performance of the SOM-MIO as regards the prediction accuracy to simulate three-dimensional subsurface flow by mirroring the experimental data which had not been included in the training. The same prediction accuracy was achieved for the solution of different inverse problems, e.g. the evaluation of the necessary water application time to arrive at a desired extension of the wetted soil volume. The striking accuracy achieved for both tasks also has to be seen in conjunction with the comparatively modest computational requirements. The SOM-MIO required only 100 seconds for its training (!), which consisted of learning 100 times a number of 1,288 input/output vectors. To perform either the complete simulation or even the entire optimization task, merely one second CPU time was needed, which contrasts with the 30 seconds required by the 1D numerical flow model for one single simulation only (!). What is more, using the SOM-MIO within a Monte Carlo analysis further proved its fundamental superiority in computational efficiency when compared to numerical simulation models.
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The presented applications of the new optimization strategy demonstrated, also in comparative applications with other current optimization tools, that GAIN-P has the potential to substantially increase irrigation efficiency by providing the necessary options for a relatively simple but nevertheless reliable, straightforward and computationally highly efficient on-field operation. The promising results of the presented alternative optimization and modelling strategy, which were exemplarily achieved for furrow and trickle irrigation, offer a strong incentive for further developing the GAIN-P methodology in order to fully satisfy the requirements for a wide application in irrigation practice.


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